



Eulerian theory of deformable media

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2016

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Eulerian theory of deformable media

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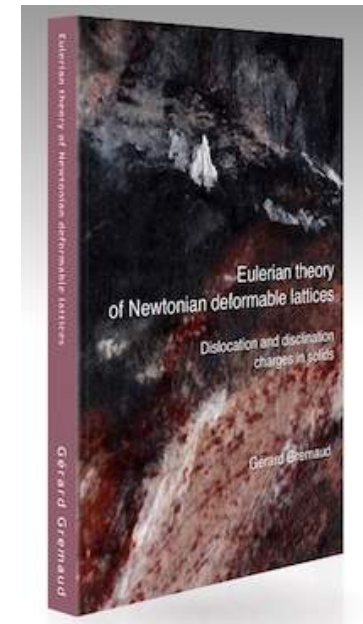
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<http://gerardgremaud.ch>



Presses Polytechniques et
Universitaires Romandes (PPUR),
Lausanne, 2013, 750 pages
(ISBN 978-2-88074-964-4)



Amazon Press,
Charleston (USA),
2016, 312 pages
(ISBN 978-2-8399-1943-2)

I - Eulerian theory of newtonian deformable medias

Coordinates systems

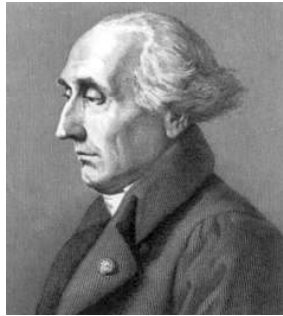
Differential geometries
(Riemann-Cartan, Finsler, Kawaguchi, ...)



Lagrange coordinates



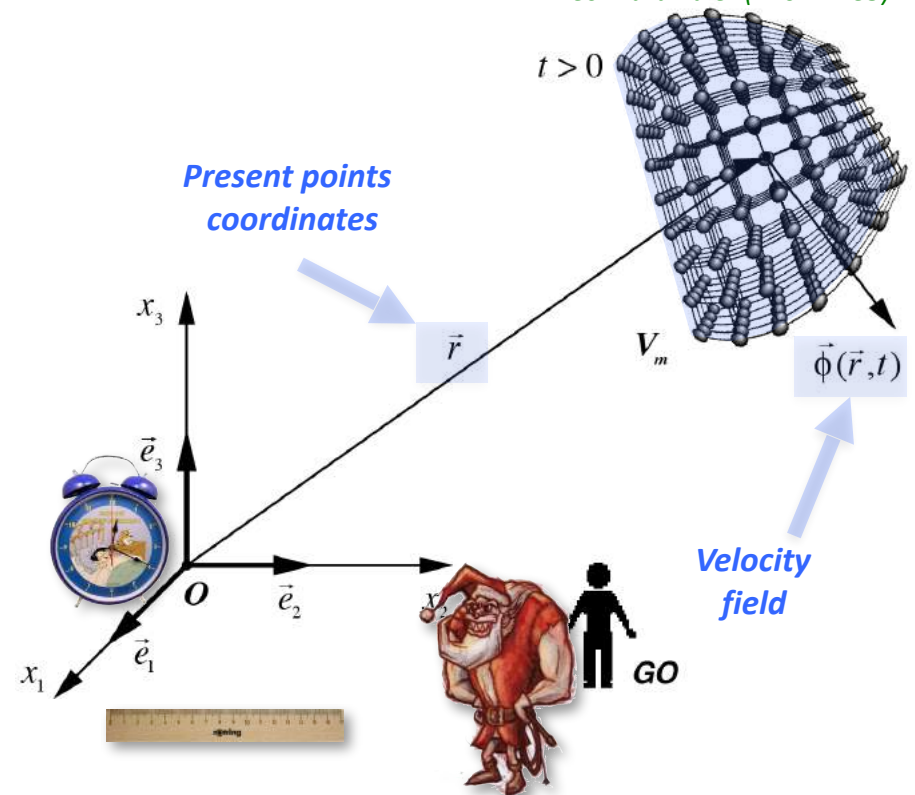
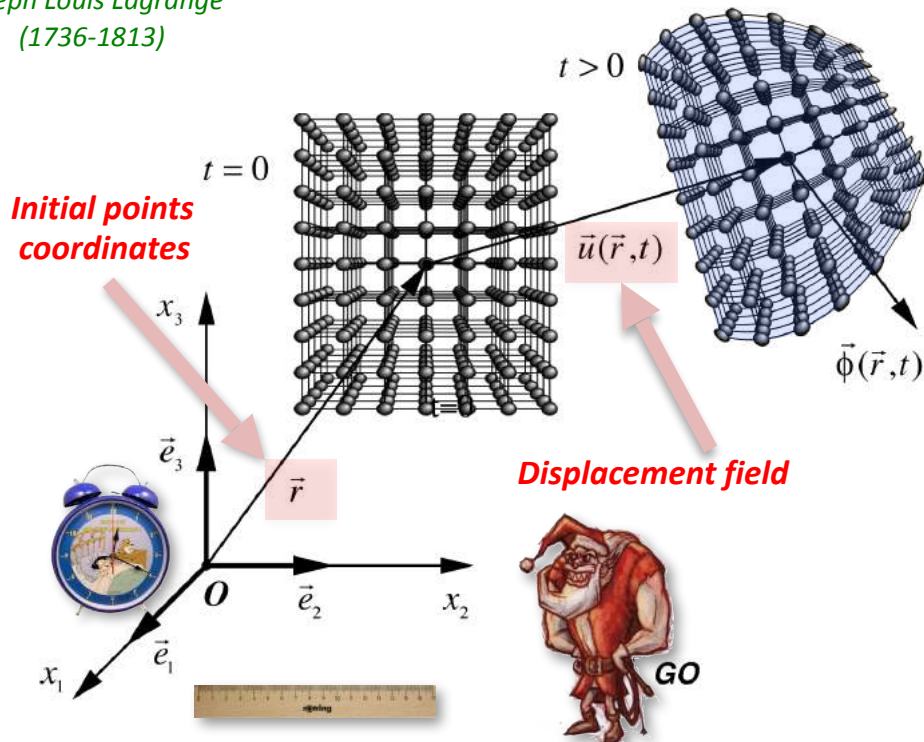
Euler coordinates



Joseph Louis Lagrange
(1736-1813)



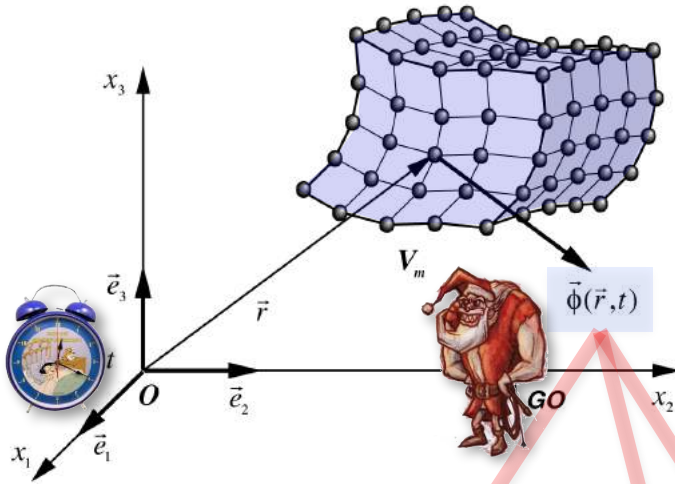
Leonhard Euler (1707-1783)



Geometrokinetic equations and distortion tensors in Euler coordinates

Temporal variations of the lattice « distortions » are linked to the spatial variations of the velocity field

Vectorial representation of the distortion tensors



Material derivative

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{\phi} \nabla)$$

Tensor of distortion

$$\frac{d\vec{\beta}_i}{dt} = -\frac{S_n}{3n} \vec{e}_i + \overline{\text{grad}} \phi_i$$

trace

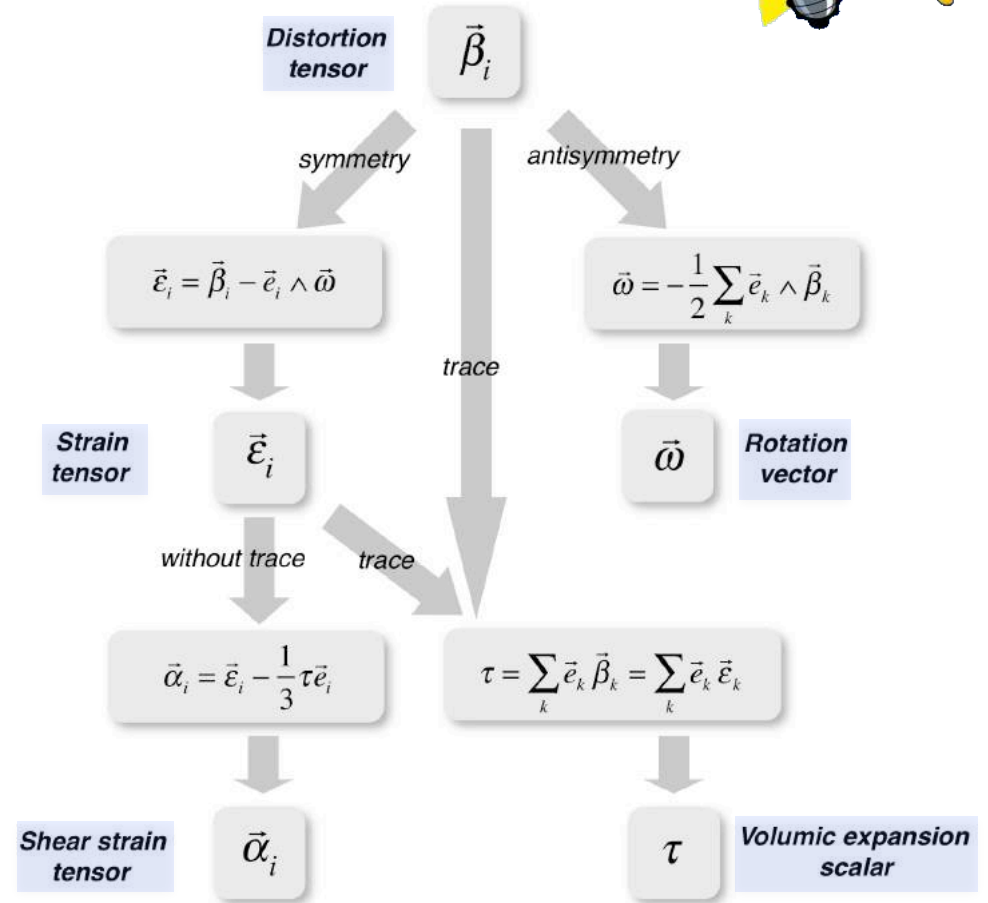
Scalar of expansion

$$\frac{d\tau}{dt} = -\frac{S_n}{n} + \text{div} \vec{\phi}$$

Antisymmetric part

Vector of rotation

$$\frac{d\vec{\omega}}{dt} = \frac{1}{2} \overline{\text{rot}} \vec{\phi}$$



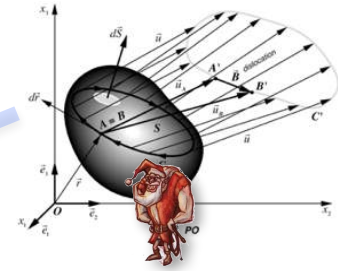
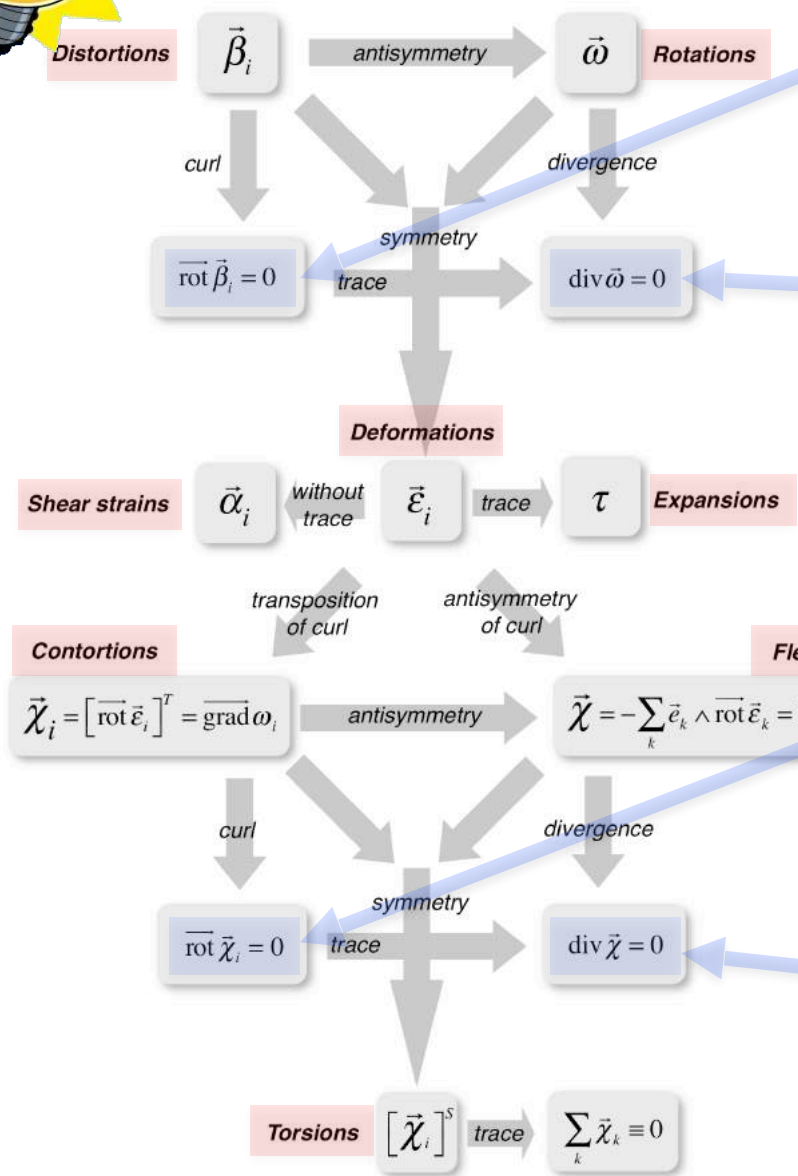
Relation with the volumic density n of sites and the volume v per site of the lattice

$$\tau = -\ln \frac{n}{n_0} = \ln \frac{v}{v_0}$$

Geometrocompatibility equations and contortion tensors in Euler coordinates

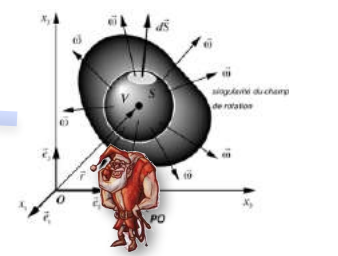


Distortion and contortion tensors and geometrocompatibility equations



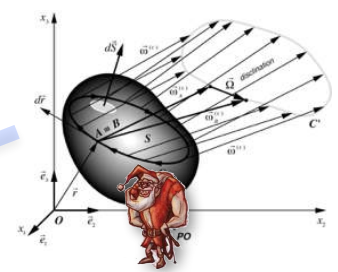
Continuity of the displacement field

$$\oint_C d\vec{u} = -\sum_k \vec{e}_k \iint_S \overline{\text{rot}} \vec{\beta}_k d\vec{S} = 0 \quad ; \quad \forall C$$



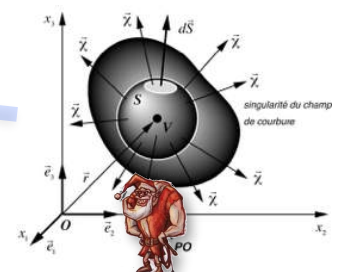
No singularity by divergence of the rotation field

$$\oiint_S \omega_{\perp} dS = \oiint_S \vec{\omega} d\vec{S} = \iiint_V \text{div } \vec{\omega} dV = 0 \quad ; \quad \forall S$$



Continuity of the rotation field associated to the deformations

$$\oint_C d\vec{\omega}^{(e)} = \sum_k \vec{e}_k \iint_S \overline{\text{rot}} \vec{\chi}_k d\vec{S} = \sum_k \vec{e}_k \iint_S \overline{\text{rot}} [\overline{\text{rot}} \vec{\epsilon}_k]^T d\vec{S} = 0$$



No singularity by divergence of the flexion field

$$\oiint_S \chi_{\perp} dS = \oiint_S \vec{\chi} d\vec{S} = \iiint_V \text{div } \vec{\chi} dV = \sum_k \vec{e}_k \iiint_V \overline{\text{rot}} \overline{\text{rot}} \vec{\epsilon}_k dV = 0$$

The only three necessary physical principles in Euler coordinates



Isaac Newton
(1643-1727)

**Axiom of
newtonian
dynamics**

$$e_{cin} = \frac{1}{2} m \vec{\phi}^2$$



**Continuity principle
for the newtonian inertial mass**

$$\frac{\partial \rho}{\partial t} = S_m - \text{div}(\rho \vec{\phi} + \vec{J}_m) = S_m - \text{div}(n \vec{p}) \quad (1)$$

**Axiom of
the first principle + kinetic energy
of thermodynamics**

$$dU = \delta W + \delta Q$$

$$e_{cin} = \frac{1}{2} m \vec{\phi}^2$$



Continuity principle for the total energy

$$n \frac{du}{dt} + n \frac{de_{cin}}{dt} = S_w^{ext} - \text{div} \vec{J}_w - \text{div} \vec{J}_q - u S_n - e_{cin} S_n \quad (2)$$



Sadi Carnot
(1799-1832)

**Axiom of
the second principle
of thermodynamics**

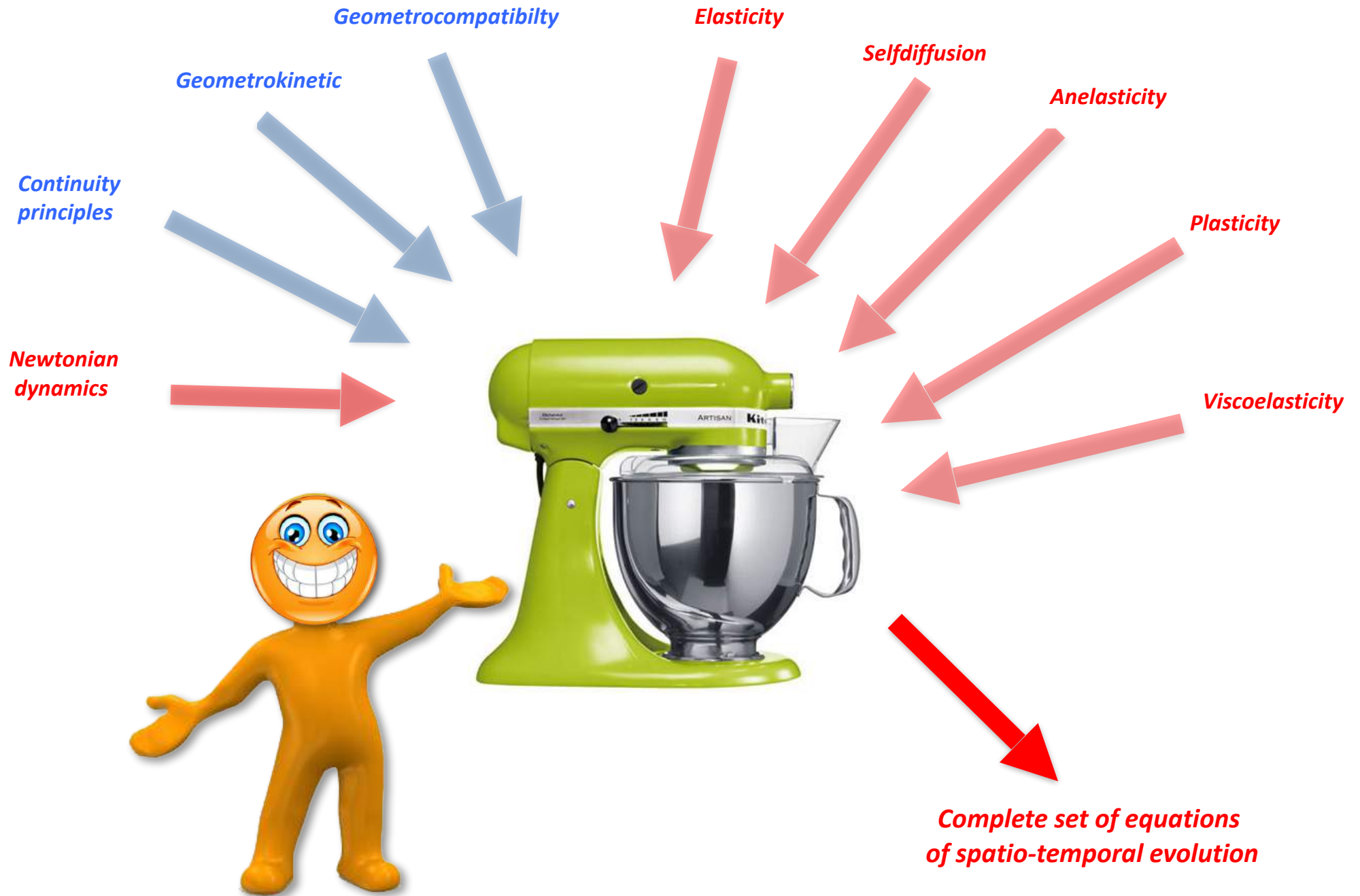
$$dS \geq \frac{\delta Q}{T}$$



Continuity principle for the entropy

$$n \frac{ds}{dt} = S_e - \text{div} \left(\frac{\vec{J}_q}{T} \right) - s S_n \quad (3)$$

Mix all these ingredients



The complete set of equations of spatio-temporal evolution in Euler coordinates

Topological equations

Fundamental equations

Heat equation



Topology

Thermic

Dynamics

Newton equation

Diffusion

Selfdiffusion equations

Equations topologiques

$$\left\{ \begin{array}{l} \frac{d\vec{\beta}_i}{dt} = \overline{\text{grad}} \phi_i \quad (1) \\ \frac{d\vec{\omega}}{dt} = \frac{1}{2} \overline{\text{rot}} \vec{\phi} \quad (2) \\ \frac{d\vec{\tau}}{dt} = \text{div} \vec{\phi} \quad (3) \end{array} \right. \quad \left\{ \begin{array}{l} \overline{\text{rot}} \vec{\beta}_i = 0 \quad (4) \\ \text{div} \vec{\omega} = 0 \quad (5) \\ d/dt = \partial / \partial t + (\vec{\phi} \nabla) \quad (6) \\ \vec{\phi} = \vec{\phi} - \vec{\phi}_0(t) - \vec{\omega}_0(t) \wedge \vec{r} \quad (7) \end{array} \right.$$

$$\left\{ \begin{array}{l} \vec{\beta}_i = \vec{\beta}_i^{(s)} + \vec{e}_i \wedge \vec{\omega}_0(t) = \vec{\beta}_i^{(s)} + \vec{\beta}_i^{(s)} + \vec{\beta}_i^{(pl)} + \vec{e}_i \wedge \vec{\omega}_0(t) \quad (8) \\ \vec{\omega} = -\frac{1}{2} \sum_k \vec{e}_k \wedge \vec{\beta}_k = \vec{\omega}^{(s)} + \vec{\omega}_0(t) = \vec{\omega}^{(s)} + \vec{\omega}^{(s)} + \vec{\omega}^{(pl)} + \vec{\omega}_0(t) \quad (9) \\ \vec{\tau} = \sum_k \vec{\beta}_k \vec{e}_k = \sum_k \vec{\beta}_k^{(s)} \vec{e}_k = \vec{\tau}^{(s)} + \vec{\tau}^{(pl)} \quad (\tau^{(s)} \equiv 0 \text{ par hypothèse}) \quad (10) \\ \vec{e}_i = \vec{\beta}_i - \vec{e}_i \wedge \vec{\omega} = \vec{\beta}_i^{(s)} - \vec{e}_i \wedge \vec{\omega}^{(s)} = \vec{e}_i^{(s)} + \vec{e}_i^{(s)} + \vec{e}_i^{(pl)} \quad (11) \\ \vec{\alpha}_i = \vec{e}_i - \frac{1}{3} \tau \vec{e}_i = \vec{\alpha}_i^{(s)} + \vec{\alpha}_i^{(s)} + \vec{\alpha}_i^{(pl)} \quad (12) \end{array} \right.$$

Equations dynamique

$$n \frac{d\vec{p}}{dt} = \rho \vec{g} + \sum_k \vec{e}_k \text{div} \vec{s}_k - \frac{1}{2} \overline{\text{rot}} \vec{m} - \overline{\text{grad}} p + n m \vec{\phi}_i \frac{dC_L}{dt} - n m \vec{\phi}_L \frac{dC_I}{dt} \quad (13)$$

$$\left\{ \begin{array}{l} n = 1 / v = n_0 \exp(-\tau^{(s)}) \quad (14) \\ \vec{p} = m(\vec{\phi} + C_I \vec{\phi}_I - C_L \vec{\phi}_L) = m\vec{\phi} + m(C_I - C_L)\vec{\phi} + \frac{m}{n}(\vec{J}_I - \vec{J}_L) \quad (15) \\ \quad \quad \quad = [\rho \vec{\phi} + m(\vec{J}_I - \vec{J}_L)] / n \\ \rho = mn(1 + C_I - C_L) \quad (16) \end{array} \right.$$

Equations de diffusion

$$\left\{ \begin{array}{l} n \frac{dC_L}{dt} = (S_{I-L} + S_L^{(pl)}) - C_L(S_L^{(pl)} - S_I^{(pl)}) - \text{div} \vec{J}_L \quad (17) \\ n \frac{dC_I}{dt} = (S_{I-L} + S_I^{(pl)}) - C_I(S_L^{(pl)} - S_I^{(pl)}) - \text{div} \vec{J}_I \quad (18) \\ \vec{J}_L = n C_L \Delta \vec{\phi}_L = n C_L (\vec{\phi}_L - \vec{\phi}) = n C_L (\vec{\phi}_L - \vec{\phi}) \quad (19) \\ \vec{J}_I = n C_I \Delta \vec{\phi}_I = n C_I (\vec{\phi}_I - \vec{\phi}) = n C_I (\vec{\phi}_I - \vec{\phi}) \quad (20) \end{array} \right.$$

Equations thermiques

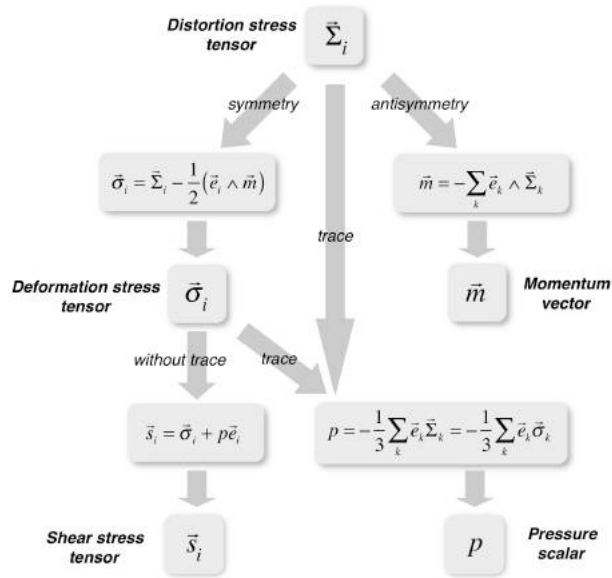
$$nT \frac{ds}{dt} = -(\mu_L^* + \mu_I^*) S_{I-L} - (\mu_L^* + h^*) S_L^{(pl)} - (\mu_I^* - h^*) S_I^{(pl)} + T \vec{J}_L \vec{X}_L \quad (21)$$

$$+ T \vec{J}_I \vec{X}_I + \vec{s}_k \frac{d\vec{\beta}_k^{(s)}}{dt} + \vec{m} \frac{d\vec{\omega}^{(s)}}{dt} + \vec{s}_k \frac{d\vec{\beta}_k^{(pl)}}{dt} + \vec{m} \frac{d\vec{\omega}^{(pl)}}{dt} - \text{div} \vec{J}_q$$

$$\left\{ \begin{array}{l} \mu_L^* = \mu_L - \frac{1}{2} m (\vec{\phi}_L^2 - 2 \Delta \vec{\phi}_L^2) \quad (22) \\ \mu_I^* = \mu_I + \frac{1}{2} m \vec{\phi}_I^2 \quad (23) \\ \vec{X}_q = \overline{\text{grad}} \frac{1}{T} \quad (24) \\ \vec{X}_L = \frac{1}{T} \left(-\overline{\text{grad}} \mu_L^* + m \frac{d}{dt} (\vec{\phi}_L - 2 \Delta \vec{\phi}_L) - m \vec{g} \right) \quad (25) \\ \vec{X}_I = \frac{1}{T} \left(-\overline{\text{grad}} \mu_I^* - m \frac{d}{dt} (\vec{\phi}_I) + m \vec{g} \right) \quad (26) \\ h^* = f + Ts + pv + \frac{1}{2} m \vec{\phi}^2 - \mu_L C_L - \mu_I C_I \quad (27) \end{array} \right.$$

The complete set of equations of spatio-temporal evolution in Euler coordinates

Phenomenological equations : state equations and dissipative equations



Distortion stress tensors



State function and equations

Fonctions et équations d'état

$f = f(\alpha_{ij}^{el}, \alpha_{ij}^{an}, \omega_k^{el}, \omega_k^{an}, \tau^{el}, C_L, C_I, T)$ (28)

$s_{ij} = n \left(\frac{\partial f}{\partial \alpha_{ij}^{el}} + \frac{\partial f}{\partial \alpha_{ji}^{el}} \right) - \frac{n}{3} \delta_{ij} \sum_k \frac{\partial f}{\partial \alpha_{kk}^{el}} = s_{ij}(\alpha_{lm}^{el}, \omega_n^{el}, \tau^{el}, \alpha_{lm}^{an}, \omega_n^{an}, C_L, C_I, T)$ (29)

$m_k = n \frac{\partial f}{\partial \omega_k^{el}} = m_k(\alpha_{lm}^{el}, \omega_n^{el}, \tau^{el}, \alpha_{lm}^{an}, \omega_n^{an}, C_L, C_I, T)$ (30)

$p = -n \frac{\partial f}{\partial \tau^{el}} = p(\alpha_{lm}^{el}, \omega_n^{el}, \tau^{el}, \alpha_{lm}^{an}, \omega_n^{an}, C_L, C_I, T)$ (31)

$s_{ij}^{dis} = n \left(\frac{\partial f}{\partial \alpha_{ij}^{an}} + \frac{\partial f}{\partial \alpha_{ji}^{an}} \right) - \frac{n}{3} \delta_{ij} \sum_k \frac{\partial f}{\partial \alpha_{kk}^{an}} = s_{ij}^{dis}(\alpha_{lm}^{el}, \omega_n^{el}, \tau^{el}, \alpha_{lm}^{an}, \omega_n^{an}, C_L, C_I, T)$ (32)

$m_k^{dis} = n \frac{\partial f}{\partial \omega_k^{an}} = m_k^{dis}(\alpha_{lm}^{el}, \omega_n^{el}, \tau^{el}, \alpha_{lm}^{an}, \omega_n^{an}, C_L, C_I, T)$ (33)

Elasticity

Anelasticity

State equations

Selfdiffusion

Anelasticity

Plasticity

Thermicity

Selfdiffusivity

+ Creation-annihilation of point defect pairs

+ Creation-annihilation of point defects

Equations de dissipation: auto-diffusion et création-annihilation de paires

$s = -\frac{\partial f}{\partial T} = s(\alpha_{lm}^{el}, \omega_n^{el}, \tau^{el}, \alpha_{lm}^{an}, \omega_n^{an}, C_L, C_I, T)$ (34)

$\mu_L = \frac{\partial f}{\partial C_L} = \mu_L(\alpha_{lm}^{el}, \omega_n^{el}, \tau^{el}, \alpha_{lm}^{an}, \omega_n^{an}, C_L, C_I, T)$ (35)

$\mu_I = \frac{\partial f}{\partial C_I} = \mu_I(\alpha_{lm}^{el}, \omega_n^{el}, \tau^{el}, \alpha_{lm}^{an}, \omega_n^{an}, C_L, C_I, T)$ (36)

Equations de dissipation: anélasticité

$\bar{J}_q = \bar{J}_q(\bar{X}_q, \bar{X}_L, \bar{X}_I, n, T, C_L, C_I, \dots)$ (37)

$\bar{J}_L = \bar{J}_L(\bar{X}_q, \bar{X}_L, \bar{X}_I, n, T, C_L, C_I, \dots)$ (38)

$\bar{J}_I = \bar{J}_I(\bar{X}_q, \bar{X}_L, \bar{X}_I, n, T, C_L, C_I, \dots)$ (39)

$S_{I-L} = S_{I-L}(\mu_L^* + \mu_I^*, n, T, C_L, C_I, \dots)$ (40)

Equations de dissipation: plasticité

$\frac{d\tau^{pl}}{dt} = \frac{S_n}{n} = \frac{1}{n}(S_L^{pl} - S_I^{pl})$ (43)

$S_L^{pl} = S_L^{pl}[(\mu_L^* + g^*), v, T, C_L, C_I, \dots]$ (44)

$S_I^{pl} = S_I^{pl}[(\mu_I^* - g^*), v, T, C_L, C_I, \dots]$ (45)

$g^* = f + pv + m\bar{\phi}^2 / 2 - \mu_L C_L - \mu_I C_I$ (46)

$\frac{d\bar{\alpha}_i^{pl}}{dt} = \frac{d\bar{\alpha}_i^{pl}}{dt}(\bar{s}_m, v, T, \dots)$ (47)

$\frac{d\bar{\omega}^{pl}}{dt} = \frac{d\bar{\omega}^{pl}}{dt}(\bar{m}, v, T, \dots)$ (48)

The complete set of equations of spatio-temporal evolution in Euler coordinates

Additional equations

<p style="text-align: center;"><i>Continuité de la masse</i></p> $\frac{\partial \rho}{\partial t} = -\operatorname{div}[\rho \vec{\phi} + m(\vec{J}_I - \vec{J}_L)] = -\operatorname{div}(n\vec{p}) \quad \text{dans } Q_{\xi_1 \xi_2 \xi_3} \quad (49)$	
<p style="text-align: center;"><i>Flux de travail et force de surface</i></p> $\left\{ \begin{array}{l} \vec{J}_w = \mu_L^* \vec{J}_L + \mu_I^* \vec{J}_I - \phi_k \bar{s}_k - \frac{1}{2}(\vec{\phi} \wedge \vec{m}) + p\vec{\phi} \quad (50) \\ \vec{F}_s = \sum_k \bar{e}_k (\bar{s}_k \cdot \vec{n}) + \frac{1}{2}(\vec{m} \wedge \vec{n}) - \vec{n}p \quad (51) \end{array} \right.$	
<p style="text-align: center;"><i>Source d'entropie</i></p> $S_e = -\frac{1}{T}(\mu_L^* + \mu_I^*)S_{I-L} - \frac{1}{T}(\mu_L^* + g^*)S_L^{pl} - \frac{1}{T}(\mu_I^* - g^*)S_I^{pl} \\ + \vec{J}_L \bar{X}_L + \vec{J}_I \bar{X}_I + \frac{1}{T} \left(\bar{s}_k^{dis} \frac{d\bar{\beta}_k^{an}}{dt} + \bar{m}^{dis} \frac{d\bar{\omega}^{an}}{dt} + \bar{s}_k \frac{d\bar{\beta}_k^{pl}}{dt} + \bar{m} \frac{d\bar{\omega}^{pl}}{dt} \right) \\ + \vec{J}_q \overline{\operatorname{grad}} \left(\frac{1}{T} \right) \quad (52)$	
<p style="text-align: center;"><i>Bilan énergétique</i></p> $n\vec{\phi} \left(\frac{d\bar{p}}{dt} - m\bar{\phi}_I \frac{dC_I}{dt} + m\bar{\phi}_L \frac{dC_L}{dt} \right) + \bar{s}_k \frac{d\bar{\beta}_k}{dt} + \bar{m} \frac{d\bar{\omega}}{dt} - p \frac{d\tau}{dt} \\ = \rho \bar{g} \vec{\phi} - \operatorname{div} \left[-\phi_k \bar{s}_k - \frac{1}{2}(\vec{\phi} \wedge \vec{m}) + p\vec{\phi} \right] \quad (53)$	

Inertial mass continuity

Work flux
Surface forces

Entropy source

Energy balance

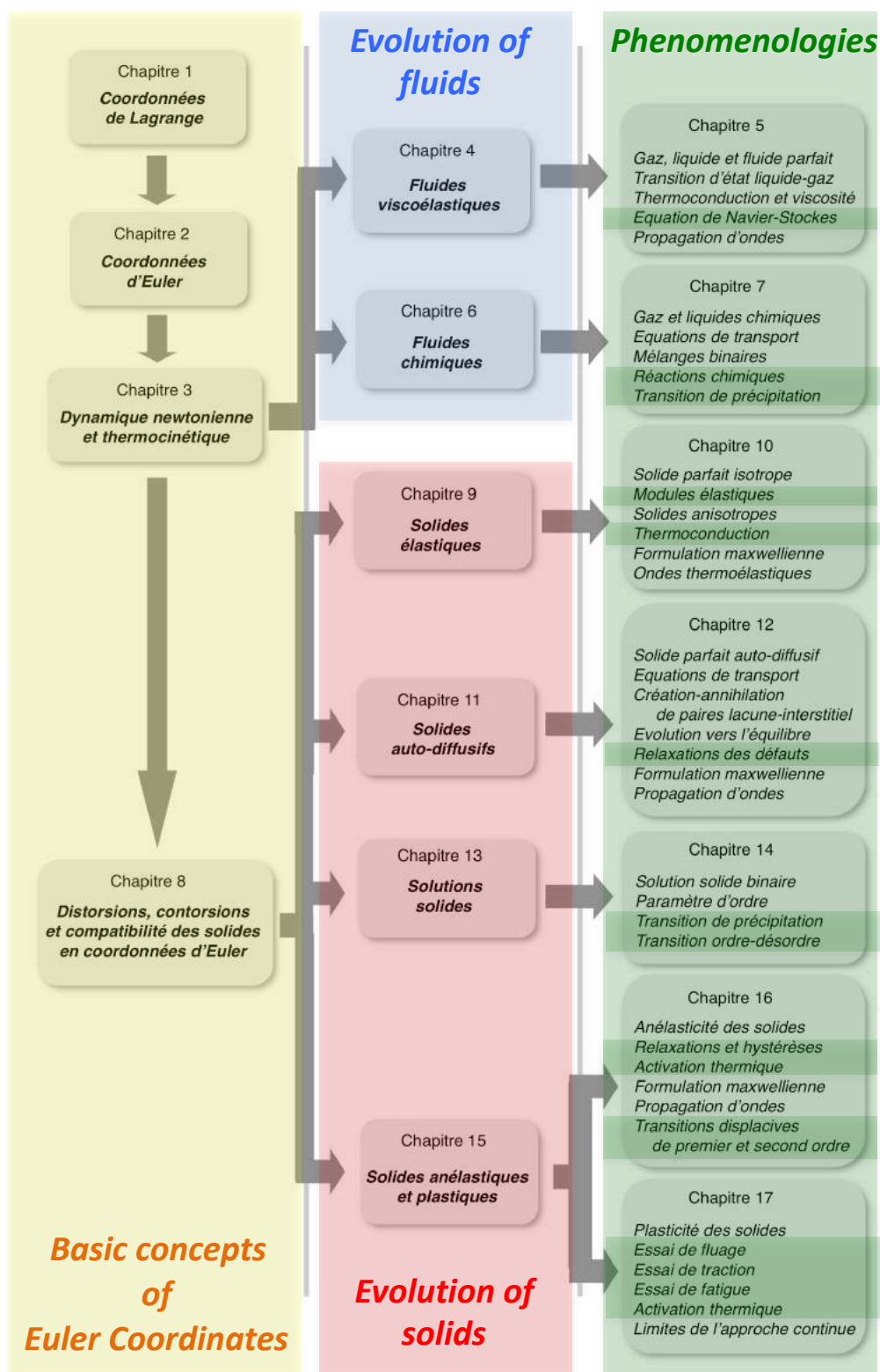


Poynting vector

II - Application: phenomenologies of usual fluids and solids



first part →



III – Dislocation and disclination charges

What's **a line** of topological singularity?

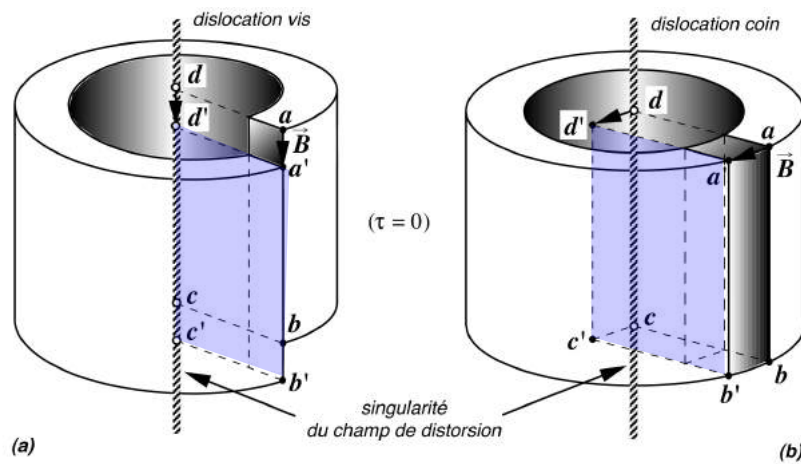


Vito Volterra
(1860-1940)

Singularity line
by translation

Screw dislocation

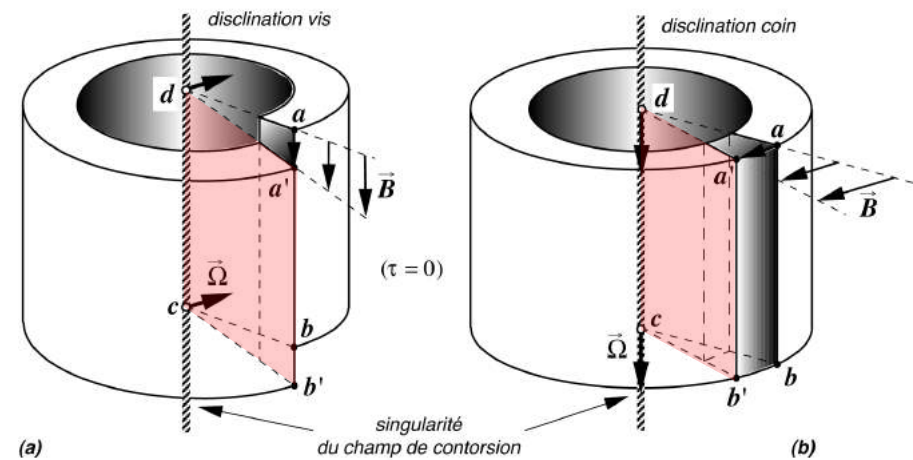
Edge dislocation



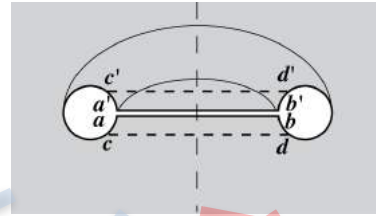
Singularity line
by rotation

Twist disclination

Wedge disclination



What's a loop of topological singularity?

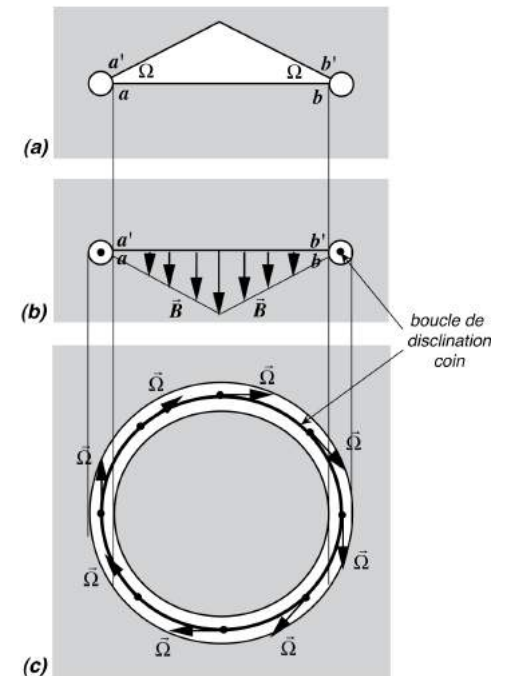
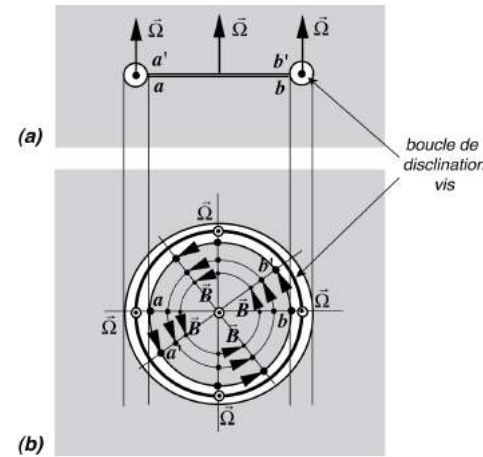
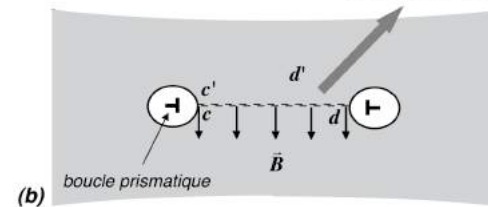
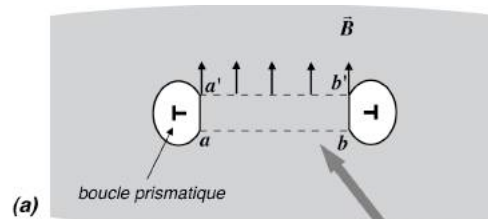
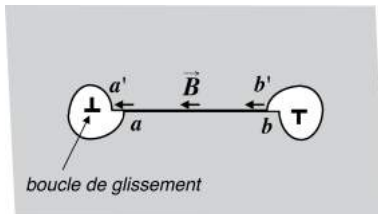


**Mixed dislocation loop
by translation**

**Edge dislocation loop
by material
addition or subtraction**

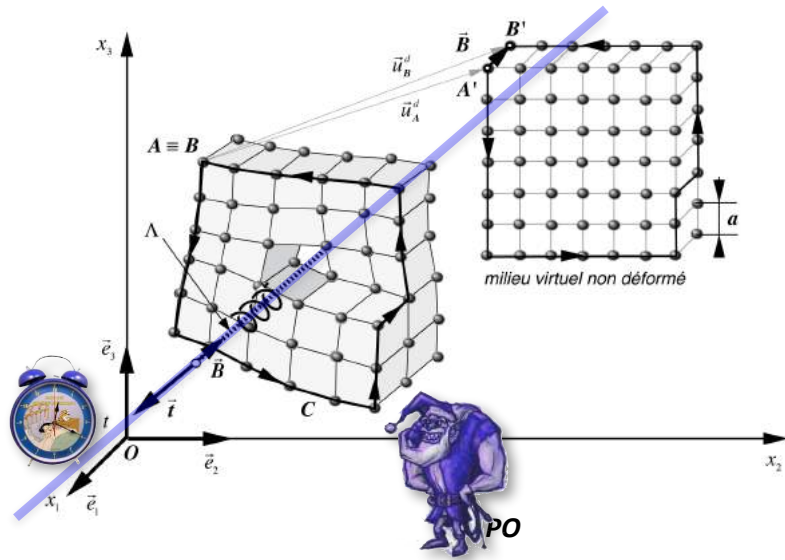
**Twist disclination loop
by rotation**

**Wedge disclination loop
by material
addition or subtraction**

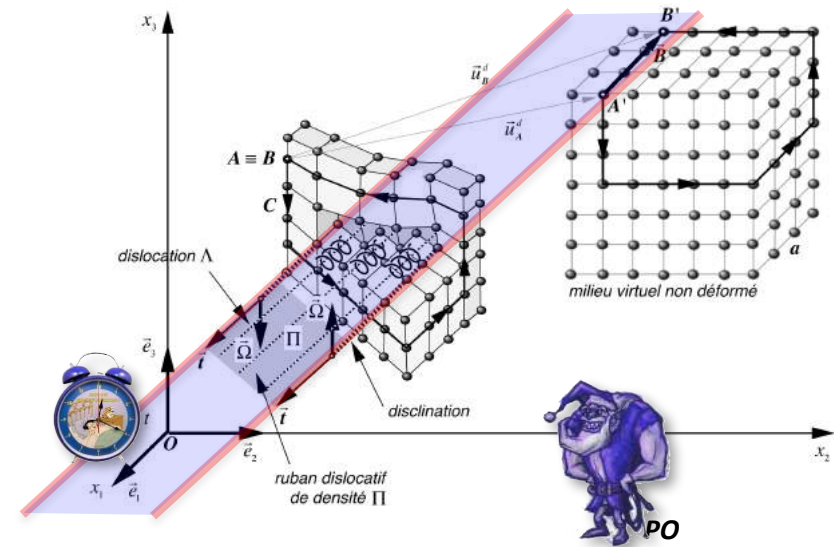


Quantification of the topological singularities as strings or membranes in solid lattices

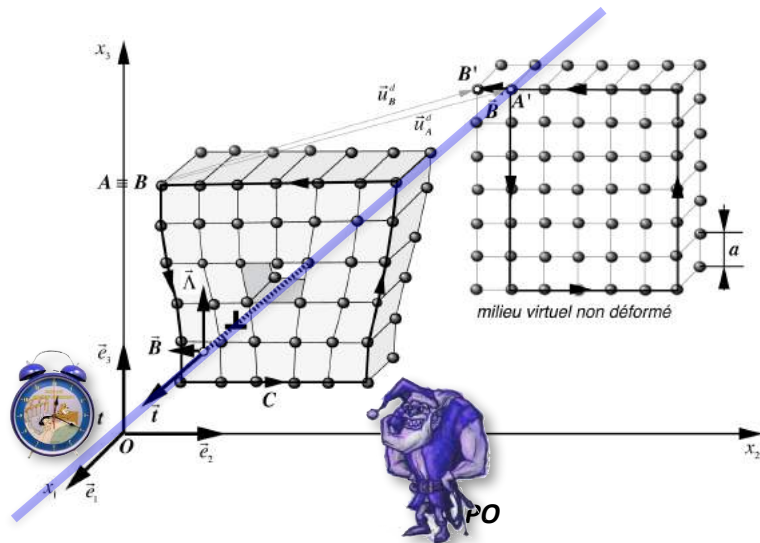
Screw dislocation string



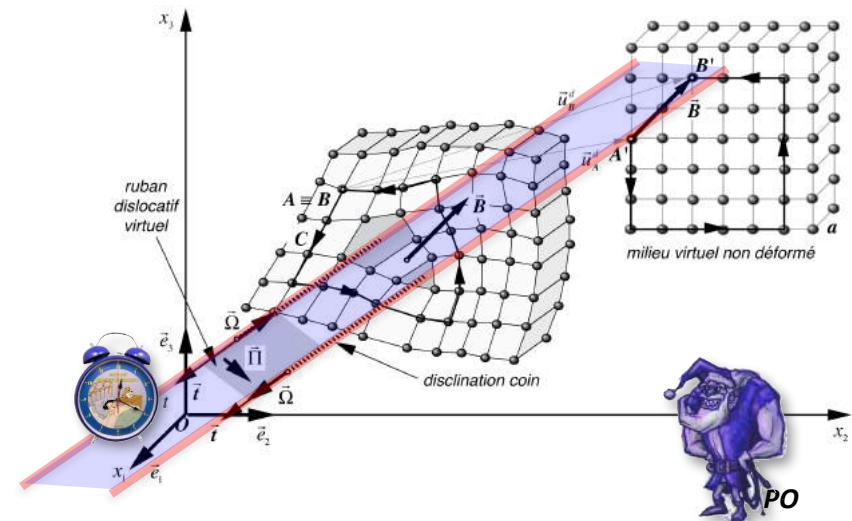
Srew dislocation membrane limited by two twist disclination strings



Edge dislocation string

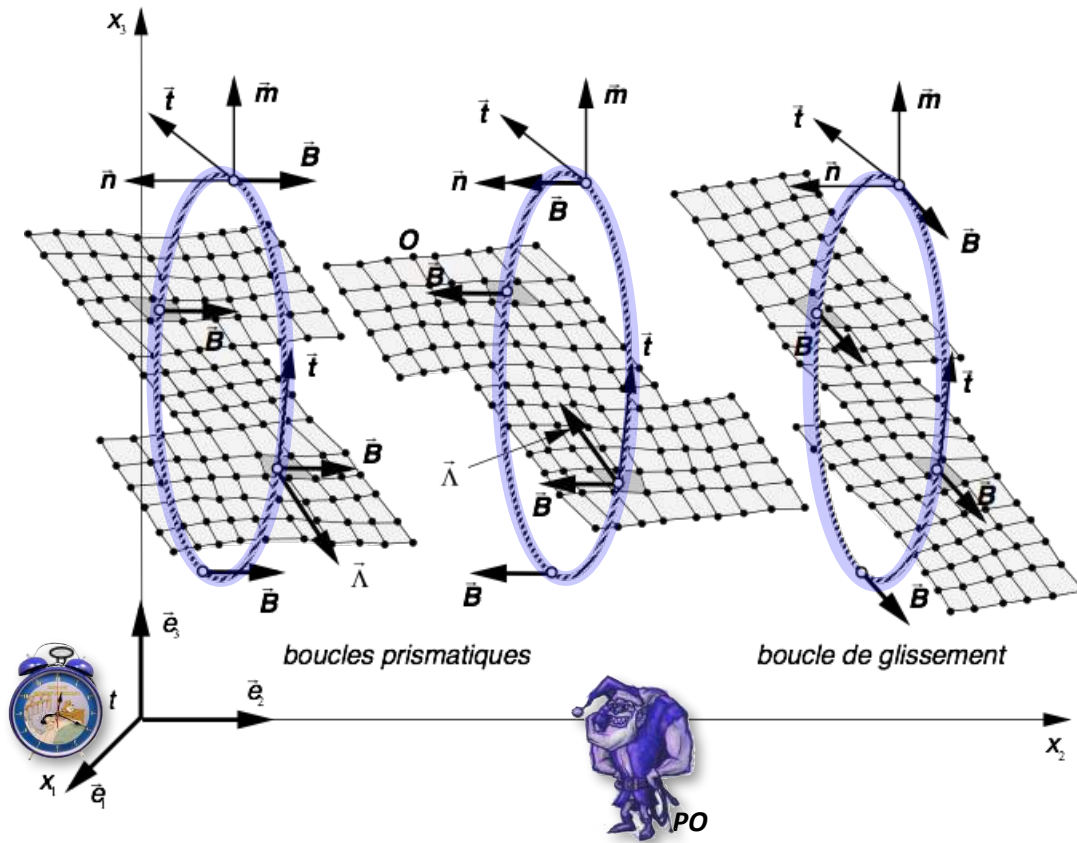


Edge dislocation membrane limited by two wedge disclination strings

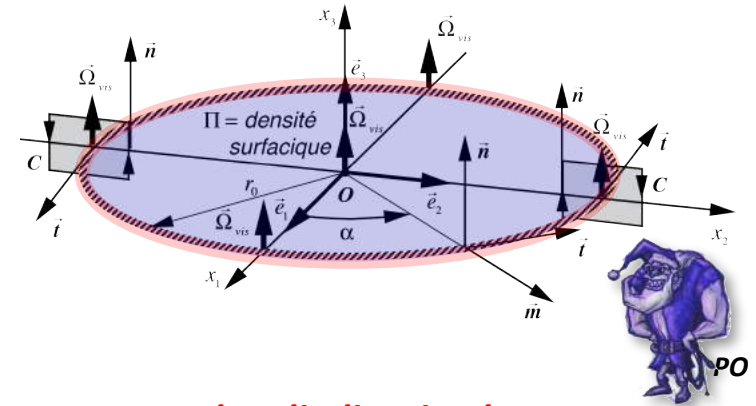


Quantification of the topological singularities as loops and membranes in solid lattices

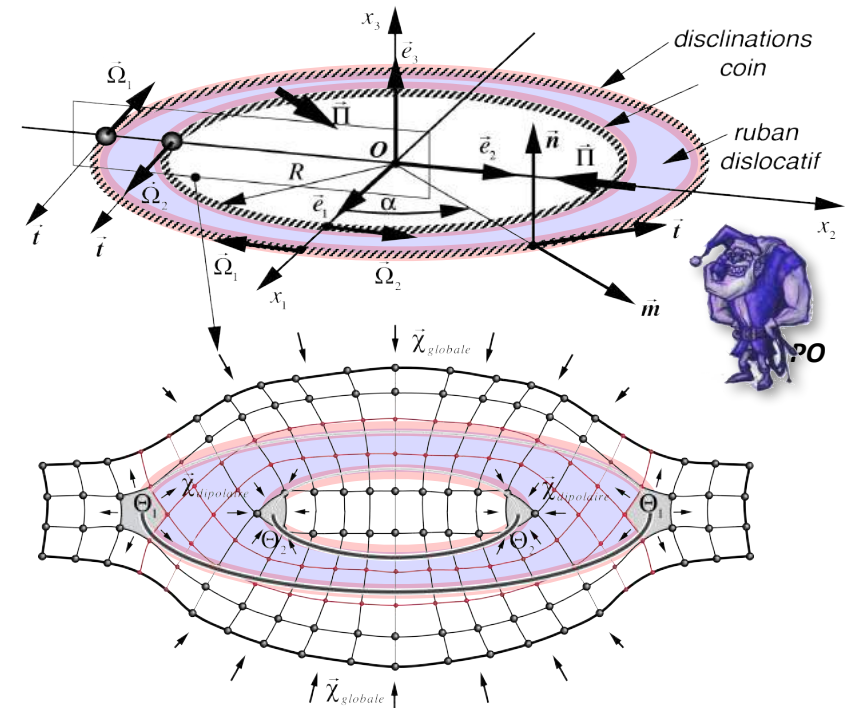
Edge and mixed dislocation loops



Twist disclination loop with screw dislocation membrane



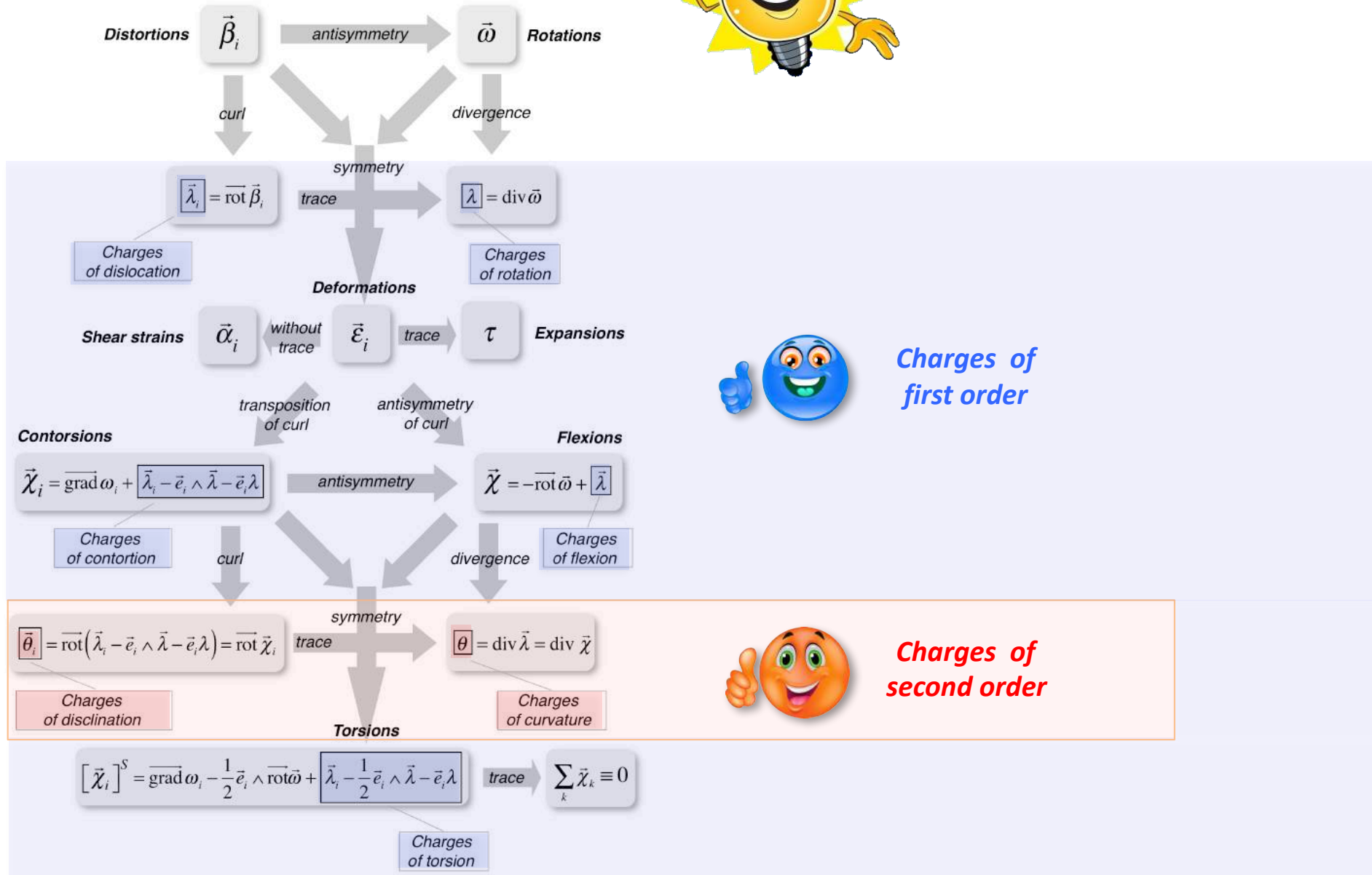
Wedge disclination loop with edge dislocation membrane



Incompatibility charges

associated to the topological singularities (strings, membranes and loops) of a solid lattice

Incompatibility equations and charge tensors



Charges of first order

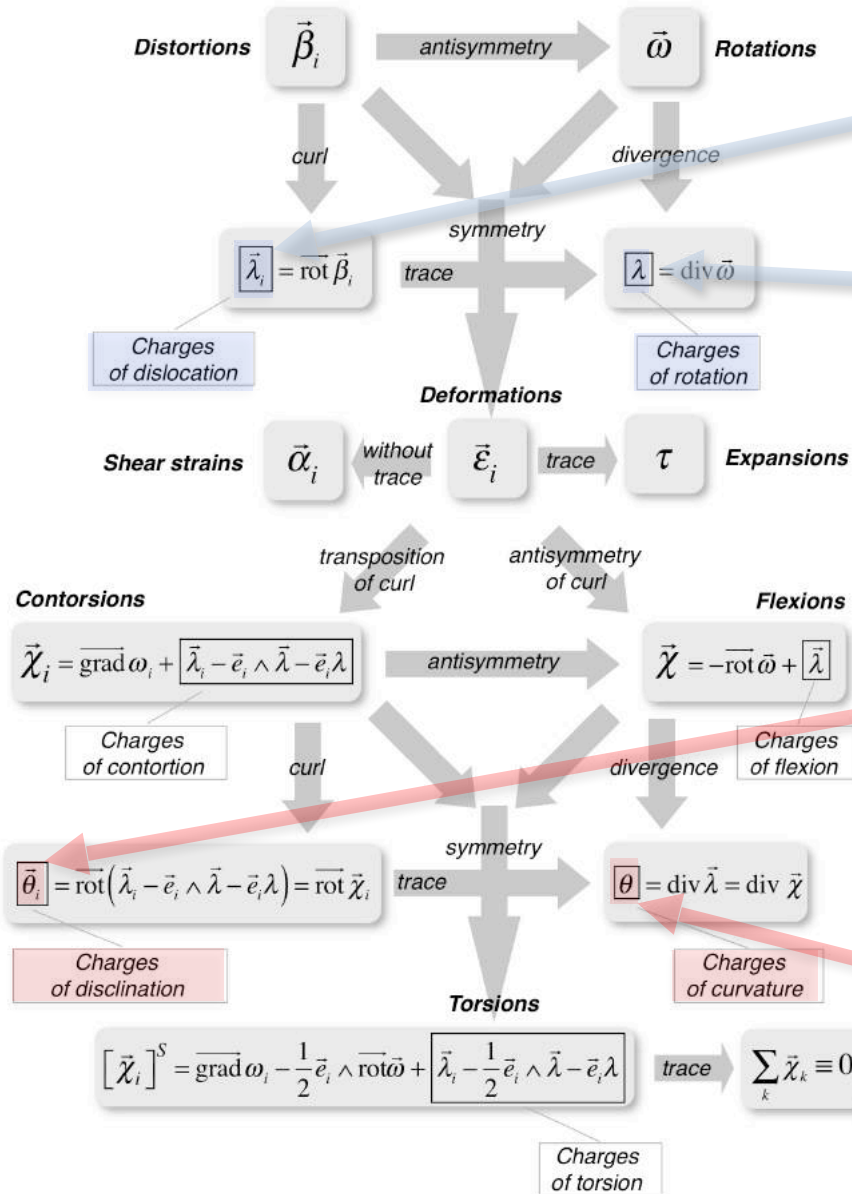


Charges of second order

Incompatibility charges

associated to the topological singularities (strings, membranes and loops) of a solid lattice

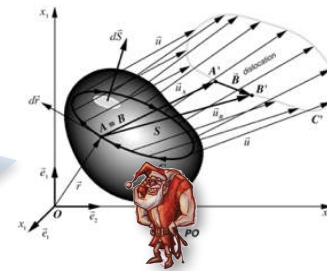
Incompatibility equations and charge tensors



Discontinuity of the displacement field

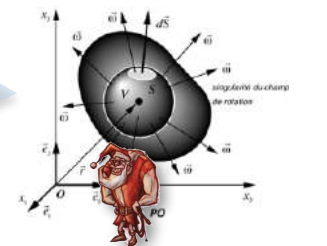
Dislocation Burgers vector

$$\vec{B} = \vec{u}_B - \vec{u}_A = \oint_C \delta \vec{u} = - \sum_k \vec{e}_k \iint_S \text{rot } \vec{\beta}_k d\vec{S} = - \sum_k \vec{e}_k \iint_S \vec{\lambda}_k d\vec{S} \neq 0$$



Singularity of the divergence of the rotation field

$$Q_\lambda = \oiint_S \vec{\omega} d\vec{S} = \iiint_V \text{div } \vec{\omega} dV = \iiint_V \vec{\lambda} dV \neq 0$$

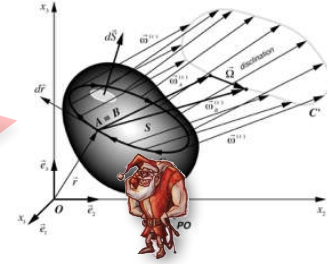


Macroscopic scalar charge of rotation

Discontinuity of the rotation field associated to the deformations

Disclination Frank vector

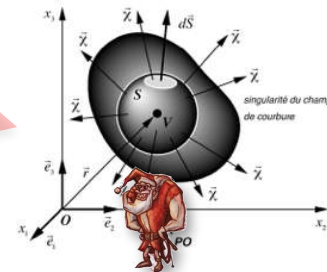
$$\vec{\Omega} = \oint_C \delta \vec{\omega}^\epsilon = \sum_k \vec{e}_k \iint_S \text{rot } \vec{\chi}_k d\vec{S} = \sum_k \vec{e}_k \iint_S \vec{\theta}_i d\vec{S} = \sum_k \vec{e}_k \oint_C (\vec{\lambda}_k - \vec{e}_k \wedge \vec{\lambda} - \vec{e}_k \lambda) d\vec{r} \neq 0$$



Singularity of the divergence of the flexion field

$$Q_\theta = \oiint_S \vec{\chi} d\vec{S} = \iiint_V \text{div } \vec{\chi} dV = \iiint_V \vec{\theta} dV \neq 0$$

Macroscopic scalar charge of flexion



The complete set of equations of spatio-temporal evolution of **a charged lattice**

Geometrokinetic
 Geometrocompatibility
 Continuity principles
 Newtonian dynamics
 Elasticity
 Anelasticity
 Selfdiffusion



Charges associated with the topological singularities



Fundamental equations

Phenomenological equations

Tableau 20.2 - Equations fondamentales d'évolution des solides auto-diffusifs, élastiques et anélastiques, avec charges plastiques dislocatives

Equations topologiques

$$\left\{ \begin{aligned} \vec{J}_i &= -\frac{d\vec{\beta}}{dt} + \text{grad} \phi_i & (1) \\ \vec{J} &= -\frac{1}{2} \sum_i \vec{e}_i \wedge \vec{J}_i = -\frac{d\vec{\omega}}{dt} + \frac{1}{2} \text{rot} \vec{\phi} & (2) \\ \frac{S_i}{n} &= \sum_i \vec{e}_i \vec{J}_i = -\frac{d\vec{\tau}}{dt} + \text{div} \vec{\phi} & (3) \end{aligned} \right. \quad \left\{ \begin{aligned} \vec{\lambda}_i &= \text{rot} \vec{\beta}_i \quad \text{avec} \quad \text{div} \vec{\lambda}_i = 0 & (4) \\ \vec{\lambda} &= -\sum_i \vec{e}_i \wedge \vec{\lambda}_i = -\sum_i \vec{e}_i \wedge \text{rot} \vec{\beta}_i & (5) \\ \vec{\lambda} &= \frac{1}{2} \sum_i \vec{e}_i \vec{J}_i = \text{div} \vec{\omega} & (6) \end{aligned} \right.$$

$$\left\{ \begin{aligned} \vec{\beta}_i &= \vec{\beta}_i^{(d)} + \vec{e}_i \wedge \vec{\omega}_i(t) = \vec{\beta}_i^{(e)} + \vec{e}_i \wedge \vec{\omega}_i(t) & (7) \\ \vec{\omega} &= -\frac{1}{2} \sum_i \vec{e}_i \wedge \vec{\beta}_i + \vec{\omega}_i(t) = \vec{\omega}^{(d)} + \vec{\omega}^{(e)} + \vec{\omega}_i(t) & (8) \\ \vec{\tau} &= \sum_i \vec{\beta}_i \vec{e}_i = \sum_i \vec{\beta}_i^{(d)} \vec{e}_i = \vec{\tau}^{(d)} \quad (\vec{\tau}^{(e)} = 0 \text{ par hypothèse}) & (9) \\ \vec{e}_i &= \vec{\beta}_i - \vec{e}_i \wedge \vec{\omega} = \vec{\beta}_i^{(d)} - \vec{e}_i \wedge \vec{\omega}^{(d)} = \vec{e}_i^{(d)} + \vec{e}_i^{(e)} & (10) \\ \vec{\alpha}_i &= \vec{e}_i - \frac{1}{3} \tau \vec{e}_i = \vec{\alpha}_i^{(d)} + \vec{\alpha}_i^{(e)} & (11) \end{aligned} \right. \quad \left\{ \begin{aligned} d \text{ (of } dt) &= \partial \text{ (of } \vec{\omega}) + (\vec{\phi} \nabla) & (12) \\ \vec{\phi} &= \vec{\phi} - \vec{\phi}_i(t) - \vec{\omega}_i(t) \wedge \vec{r} & (13) \end{aligned} \right.$$

Equations dynamique

$$\left\{ \begin{aligned} n \frac{d\vec{p}}{dt} &= \rho \vec{g} + \sum_i \vec{e}_i \text{div} \vec{J}_i - \frac{1}{2} \text{rot} \vec{\omega} - \text{grad} \rho + \sum_i \vec{e}_i \text{div} \vec{S}_i - n m \vec{\omega}_i \frac{dC_i}{dt} & (14) \\ n &= 1/v = n_i e^{e^*} & (15) \\ \vec{p} &= m(\vec{\phi} + C_i \vec{e}_i - C_i \vec{\phi}_i) - n \vec{\omega} + m(C_i - C_i) \vec{\phi} + \frac{m}{n} (J_i - J_i) = \frac{1}{n} [\rho \vec{\phi} + m(J_i - J_i)] & (16) \\ \rho &= mn(1 + C_i - C_i) & (17) \end{aligned} \right.$$

Equations de diffusion

$$\left\{ \begin{aligned} n \frac{dC_i}{dt} &= (S_{i-1} + S_i^{(e)}) - C_i (S_i^{(e)} - S_i^{(d)}) - \text{div} \vec{J}_i & (18) \\ n \frac{dC_i}{dt} &= (S_i - S_i^{(e)}) - C_i (S_i^{(e)} - S_i^{(d)}) - \text{div} \vec{J}_i & (19) \\ \vec{J}_i &= n \Delta \phi_i = n C_i (\vec{\phi}_i - \vec{\phi}) = n C_i (\vec{\phi}_i - \vec{\phi}) & (20) \\ \vec{J}_i &= n C_i \Delta \phi_i = n C_i (\vec{\phi}_i - \vec{\phi}) = n C_i (\vec{\phi}_i - \vec{\phi}) & (21) \end{aligned} \right.$$

Equations thermiques

$$\left\{ \begin{aligned} nT \frac{d\vec{s}}{dt} &= -(\mu_i' + \mu_i'') S_{i-1} - (\mu_i' + n') S_i' - (\mu_i' - n') S_i'' + T \vec{J}_i \vec{X}_i & (22) \\ &+ T \vec{J}_i \vec{X}_i + s_i^{(d)} \frac{d\vec{\beta}_i^{(d)}}{dt} + m^{(d)} \frac{d\vec{\omega}^{(d)}}{dt} + s_i \vec{J}_i + m \vec{J} - \text{div} \vec{J}_i & (23) \end{aligned} \right. \quad \left\{ \begin{aligned} \mu_i' &= \mu_i - \frac{1}{2} m (\vec{\phi}_i^2 - 2\Delta \phi_i^2) & (24) \\ \mu_i'' &= \mu_i + \frac{1}{2} m \vec{\phi}_i^2 & (25) \\ \vec{X}_i &= \text{grad} \frac{d\vec{\beta}_i}{dt} & (26) \\ \vec{X}_i &= \frac{1}{T} \left(-\text{grad} \mu_i' + m \frac{d}{dt} (\vec{\phi}_i - 2\Delta \phi_i) - m \vec{g} \right) & (27) \\ \vec{X}_i &= \frac{1}{T} \left(-\text{grad} \mu_i'' - m \frac{d}{dt} (\vec{\phi}_i) + m \vec{g} \right) & (28) \\ h' &= f + T s + p v + \frac{1}{2} m \vec{\phi}^2 - \mu_i C_i - \mu_i C_i & (29) \end{aligned} \right.$$

Equations liées aux charges

$$\left\{ \begin{aligned} \frac{d\vec{\lambda}_i}{dt} &= S_i^{(d)} - (\vec{v} \nabla) \vec{\lambda}_i = S_i^{(d)} + \text{rot} (\vec{v} \wedge \vec{\lambda}_i) = S_i^{(d)} - \text{rot} \vec{J}_i & (30) \\ \frac{d\vec{\lambda}}{dt} &= S^{(d)} - (\vec{v} \nabla) \vec{\lambda} = S^{(d)} + \text{rot} (\vec{v} \wedge \vec{\lambda}) - \vec{v} \text{div} \vec{\lambda} = S^{(d)} - 2 \text{rot} \vec{J}^{(d)} - \vec{\rho} \vec{\rho} & (31) \\ \vec{J}_i &= \vec{e}_i \wedge \vec{S}_i & (32) \\ \vec{S}_i &= \frac{1}{2} \sum_i \vec{e}_i \wedge \vec{S}_i & (33) \\ \vec{S}_i &= \frac{1}{2} \sum_i \vec{e}_i \vec{S}_i & (34) \\ S^{(d)} &= -\sum_i \vec{e}_i \wedge S_i^{(d)} & (35) \\ S^{(e)} &= \frac{1}{2} \sum_i \vec{e}_i S_i^{(e)} & (36) \\ \vec{J}_{rc} &= \sum_i (\vec{e}_i \wedge \vec{S}_i) + \lambda \vec{m} + \frac{1}{2} (m \wedge \vec{\lambda}) + \vec{\lambda} p + \vec{v} \wedge \vec{A} & (37) \end{aligned} \right.$$

Equations phénoménologiques d'évolution des solides auto-diffusifs, élastiques et anélastiques, avec charges plastiques dislocatives

Fonctions et équations d'état

$$\left\{ \begin{aligned} f &= f(\alpha_i^{(d)}, \alpha_i^{(e)}, \omega_i^{(d)}, \omega_i^{(e)}, \tau^{(d)}, C_i, C_i, T) & (38) \\ s_i &= \frac{n}{2} \left(\frac{\partial f}{\partial \alpha_i^{(d)}} + \frac{\partial f}{\partial \alpha_i^{(e)}} \right) - \frac{n}{3} \delta_i \sum_j \frac{\partial f}{\partial \alpha_j^{(d)}} = s_i(\alpha_i^{(d)}, \omega_i^{(d)}, \tau^{(d)}, \alpha_i^{(e)}, \omega_i^{(e)}, C_i, C_i, T) & (39) \\ m_i &= n \frac{\partial f}{\partial \omega_i^{(d)}} = m_i(\alpha_i^{(d)}, \omega_i^{(d)}, \tau^{(d)}, \alpha_i^{(e)}, \omega_i^{(e)}, C_i, C_i, T) & (40) \\ p &= -n \frac{\partial f}{\partial \tau^{(d)}} = p(\alpha_i^{(d)}, \omega_i^{(d)}, \tau^{(d)}, \alpha_i^{(e)}, \omega_i^{(e)}, C_i, C_i, T) & (41) \\ s_i^{(d)} &= \frac{n}{2} \left(\frac{\partial f}{\partial \alpha_i^{(d)}} + \frac{\partial f}{\partial \alpha_i^{(e)}} \right) - \frac{n}{3} \delta_i \sum_j \frac{\partial f}{\partial \alpha_j^{(d)}} = s_i^{(d)}(\alpha_i^{(d)}, \omega_i^{(d)}, \tau^{(d)}, \alpha_i^{(e)}, \omega_i^{(e)}, C_i, C_i, T) & (42) \\ m_i^{(d)} &= n \frac{\partial f}{\partial \omega_i^{(d)}} = m_i^{(d)}(\alpha_i^{(d)}, \omega_i^{(d)}, \tau^{(d)}, \alpha_i^{(e)}, \omega_i^{(e)}, C_i, C_i, T) & (43) \\ s &= -n \frac{\partial f}{\partial T} = s(\alpha_i^{(d)}, \omega_i^{(d)}, \tau^{(d)}, \alpha_i^{(e)}, \omega_i^{(e)}, C_i, C_i, T) & (44) \\ \mu_i &= \frac{\partial f}{\partial C_i} = \mu_i(\alpha_i^{(d)}, \omega_i^{(d)}, \tau^{(d)}, \alpha_i^{(e)}, \omega_i^{(e)}, C_i, C_i, T) & (45) \\ \mu_i &= \frac{\partial f}{\partial C_i} = \mu_i(\alpha_i^{(d)}, \omega_i^{(d)}, \tau^{(d)}, \alpha_i^{(e)}, \omega_i^{(e)}, C_i, C_i, T) & (46) \end{aligned} \right.$$

Equations de dissipation: auto-diffusion et création/annihilation de paires

$$\left\{ \begin{aligned} \vec{J}_i &= \vec{J}_i(\vec{X}_i, \vec{X}_i, \vec{X}_i, n, T, C_i, C_i, \dots) & (47) \\ \vec{J}_i &= \vec{J}_i(\vec{X}_i, \vec{X}_i, \vec{X}_i, n, T, C_i, C_i, \dots) & (48) \\ \vec{J}_i &= \vec{J}_i(\vec{X}_i, \vec{X}_i, \vec{X}_i, n, T, C_i, C_i, \dots) & (49) \\ S_{i-1} &= S_{i-1}(\mu_i, n, T, C_i, C_i, \dots) & (50) \end{aligned} \right.$$

Equations de dissipation: anélasticité

$$\left\{ \begin{aligned} \vec{s}_i &= s_i^{(d)}(\vec{\omega}_i, v, T, \dots) + s_i^{(e)}(\vec{\omega}_i, v, T, \dots) & (51) \\ \vec{m} &= \vec{m}^{(d)}(\vec{\omega}_i, v, T, \dots) + \vec{m}^{(e)}(\vec{\omega}_i, v, T, \dots) & (52) \end{aligned} \right.$$

Equations de dissipation: flux de charges plastiques dislocatives

$$\left\{ \begin{aligned} \vec{J}_i &= \vec{e}_i \wedge \vec{v} = \vec{J}_i(\vec{e}_i, \vec{e}_i, v, T, \dots) & (53) \\ \vec{J} &= -\frac{1}{2} \sum_i \vec{e}_i \wedge \vec{J}_i = \vec{J} \vec{v} + \frac{1}{2} (\vec{\lambda} \wedge \vec{v}) = \vec{J}(\vec{m}, \vec{\lambda}, v, T, \dots) & (54) \\ \frac{S_i}{n} &= \sum_i \vec{e}_i \vec{J}_i = -\vec{J} \vec{v} = \frac{1}{n} (S_i^{(d)} - S_i^{(e)}) & (55) \\ S_i^{(d)} &= S_i^{(d)}[(\mu_i' + s'), p, \vec{\lambda}, v, T, C_i, C_i, \dots] & (56) \\ S_i^{(e)} &= S_i^{(e)}[(\mu_i' - s'), p, \vec{\lambda}, v, T, C_i, C_i, \dots] & (57) \\ g' &= f + p v + n \frac{1}{2} \sum_i \vec{e}_i \cdot \mu_i C_i & (58) \end{aligned} \right.$$

Equations de dissipation: sources de charges plastiques dislocatives

$$\left\{ \begin{aligned} S_i^{(d)} &= S_i^{(d)}(\dots) & (59) \\ S_i^{(e)} &= S_i^{(e)}(\dots) & (60) \end{aligned} \right. \Rightarrow \left\{ \begin{aligned} S_i^{(d)} &= -\sum_i \vec{e}_i \wedge S_i^{(d)} = S_i^{(d)}(\dots) & (61) \\ S_i^{(e)} &= \frac{1}{2} \sum_i \vec{e}_i S_i^{(e)} = S_i^{(e)}(\dots) & (62) \end{aligned} \right.$$

Equations additionnelles d'évolution

Continuité de la masse

$$\frac{\partial \rho}{\partial t} = -\text{div}[\rho \vec{v} + m(\vec{J}_i - J_i)] = -\text{div}(n \vec{p}) \quad \text{dans } \mathcal{Q}_{S_0}^E \vec{e}_i \vec{e}_i \vec{e}_i \quad (63)$$

Flux de travail et force de surface

$$\left\{ \begin{aligned} \vec{J}_i &= \mu_i \vec{J}_i + \mu_i' \vec{J}_i - \vec{e}_i \vec{S}_i - \frac{1}{2} (\vec{\phi} \wedge \vec{m}) + p \vec{e}_i & (64) \\ \vec{F}_i &= \sum_i \vec{e}_i (\vec{e}_i \cdot \vec{m}) + \frac{1}{2} (\vec{m} \wedge \vec{h}) - \vec{h} p & (65) \end{aligned} \right.$$

Source d'entrainement

$$S_i = -\frac{1}{2} (\mu_i' + \mu_i'') S_{i-1} - \frac{1}{2} (\mu_i' - \mu_i'') S_i' - \frac{1}{2} (\mu_i' - s') S_i'' + \vec{J}_i \vec{X}_i + \vec{J}_i \vec{X}_i + \frac{1}{2} \left(\frac{d\vec{\beta}_i^{(d)}}{dt} + m^{(d)} \frac{d\vec{\omega}^{(d)}}{dt} + s_i \vec{J}_i + m \vec{J} + \vec{J}_i \text{grad} \left(\frac{1}{T} \right) \right) \quad (66)$$

Bilan énergétique

$$\left\{ \begin{aligned} n \vec{\omega} \left(\frac{d\vec{p}}{dt} - m \vec{\omega} \frac{dC_i}{dt} + m \vec{\omega} \frac{dC_i}{dt} \right) + \left(\vec{s}_i \frac{d\vec{\beta}_i}{dt} + \vec{m} \frac{d\vec{\omega}}{dt} - p \frac{d\tau}{dt} \right) & (67) \\ + \left(\vec{s}_i \vec{J}_i + m \vec{J} - p \frac{S_i}{n} \right) = \rho \vec{\omega} \vec{\omega} - \text{div} \left(-\vec{\phi} \vec{S}_i - \frac{1}{2} (\vec{\phi} \wedge \vec{m}) + p \vec{e}_i \right) & (68) \end{aligned} \right.$$

Additional equations

Topology
 Dynamics
 Selfdiffusion

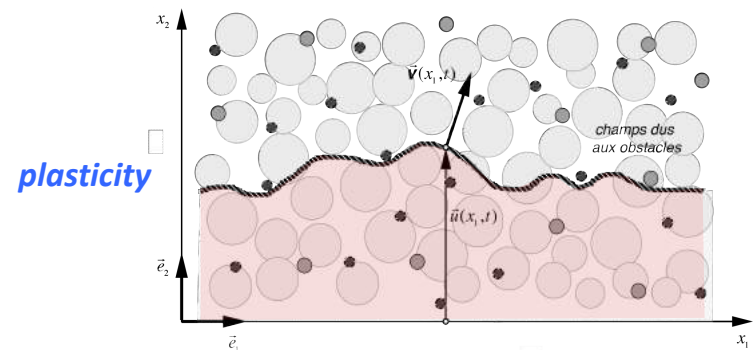
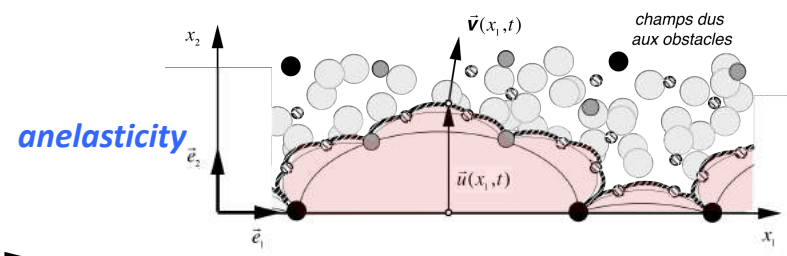
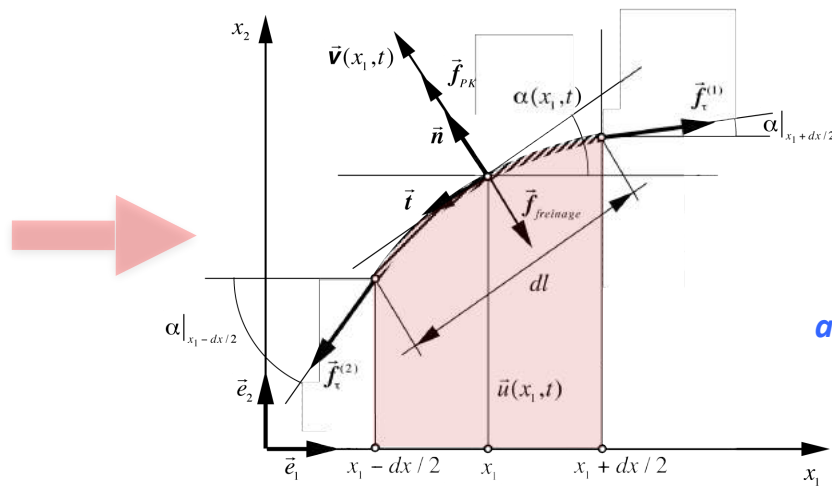
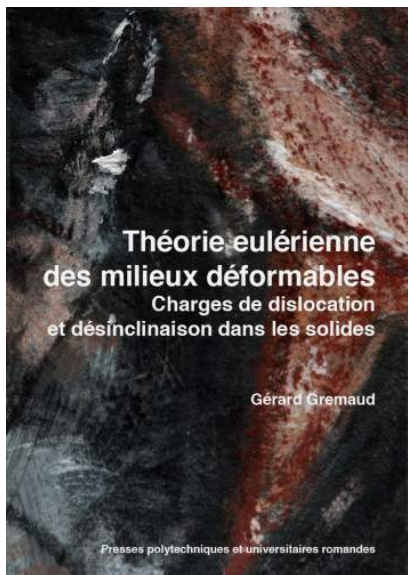
Thermics
 Charges due to the topological singularities

State equations
 Selfdiffusion
 Anelasticity

Flux and sources of charges
 Additional equations

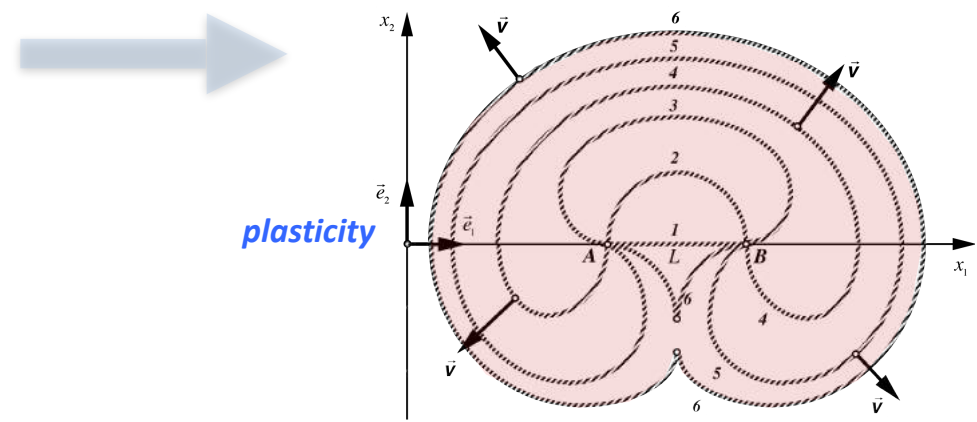
IV - Application: elements of dislocation theory in usual solids

String model of a dislocation line

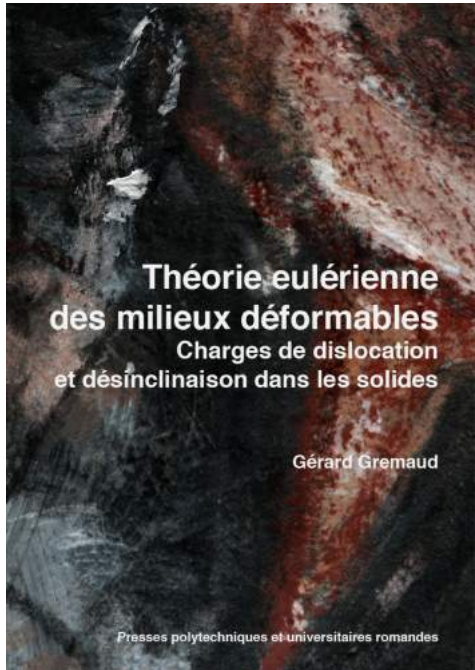


$$\left(M_0 \frac{\partial^2 u}{\partial t^2} + B_f \frac{\partial u}{\partial t} \right) \sqrt{1 + (\partial u / \partial x_1)^2} - \frac{\partial}{\partial x_1} \left[\tau \frac{\partial u}{\partial x_1} / \sqrt{1 + (\partial u / \partial x_1)^2} \right]$$

$$= B s_{23} + \sum_{n=1}^N f_n(x_1, u(x_1, t)) + F_{fluctuation}(x_1, u(x_1, t), t)$$



Other consequences



Relativistic dynamics of the charges

Maxwell equations and Lorentz force at constant volumic expansion

Interactions of electrical type and of gravitational type between charges

String model of the dislocation line

+

Absence of particles analogue to magnetic monopoles

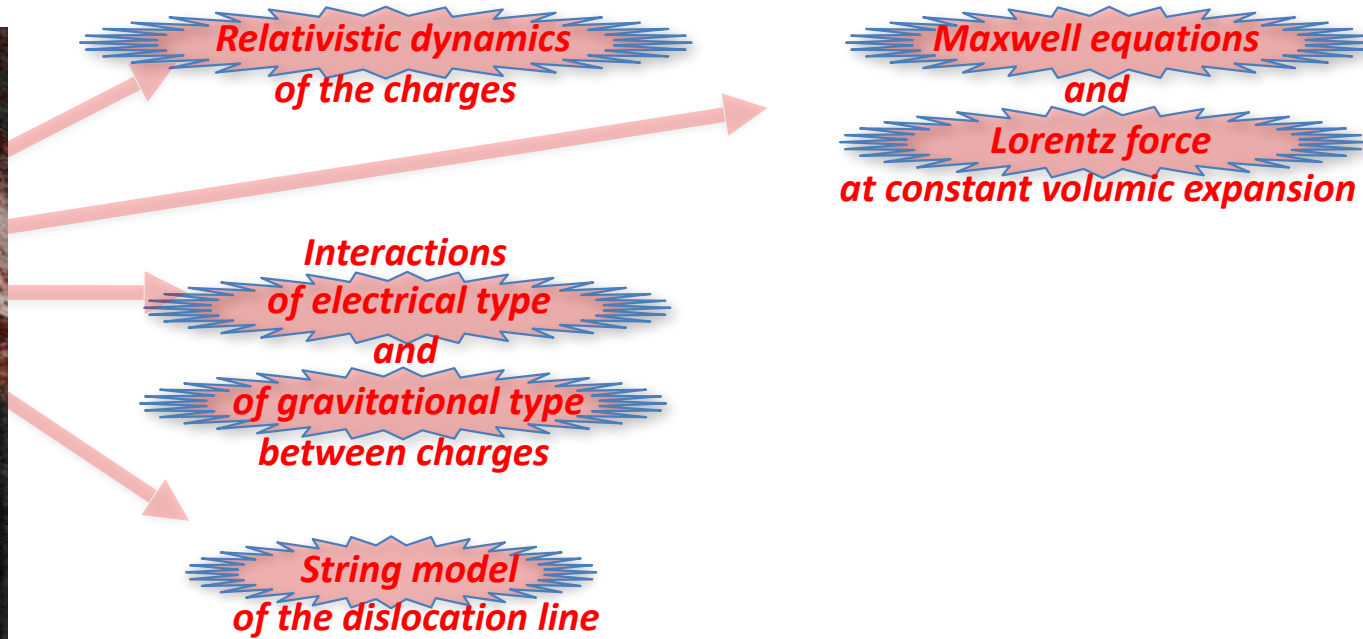
Possible solution of the famous paradox of the electron field energy

Existence of a small asymmetry between curvature charges of vacancy or interstitial type

$$\begin{aligned}
 & \left\{ \begin{array}{l} -\frac{d(2\vec{\omega})}{dt} + \overline{\text{rot}} \vec{\phi} = (2\vec{J}), \\ \text{div}(2\vec{\omega}) = (2\lambda), \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} -\frac{\partial \vec{D}}{\partial t} + \overline{\text{rot}} \vec{H} = \vec{j} \\ \text{div} \vec{D} = \rho \end{array} \right. \\
 & \left\{ \begin{array}{l} \frac{d(n\vec{p})}{dt} = -\overline{\text{rot}} \left(\frac{\vec{m}}{2} \right), \\ \text{div}(n\vec{p}) = 0 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} \frac{\partial \vec{B}}{\partial t} = -\overline{\text{rot}} \vec{E} \\ \text{div} \vec{B} = 0 \end{array} \right. \\
 & \left\{ \begin{array}{l} (2\vec{\omega}) = \left(\frac{1}{nk_2} \right) \left(\frac{\vec{m}}{2} \right) + (2\vec{\omega}^{an}) + (2\vec{\omega}_0(t)), \\ (n\vec{p}) = (nm) \left[\vec{\phi} + (C_I - C_L) \vec{\phi} + \left(\frac{1}{n} (\vec{J}_I - \vec{J}_L) \right) \right] \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} \vec{D} = \epsilon_0 \vec{E} + \vec{P} + \vec{P}_0(t) \\ \vec{B} = \mu_0 \left[\vec{H} + (\chi^{para} + \chi^{dia}) \vec{H} + \vec{M} \right] \end{array} \right. \\
 & \left\{ \begin{array}{l} \frac{d(2\lambda)}{dt} = -\text{div}(2\vec{J}), \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} \frac{\partial \rho}{\partial t} = -\text{div} \vec{j} \end{array} \right. \\
 & \left\{ \begin{array}{l} -\left(\frac{\vec{m}}{2} \right) (2\vec{J}) = \\ \vec{\phi} \frac{d(n\vec{p})}{dt} + \left(\frac{\vec{m}}{2} \right) \frac{d(2\vec{\omega})}{dt} - \text{div} \left(\vec{\phi} \wedge \left(\frac{\vec{m}}{2} \right) \right) \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} -\vec{E} \vec{j} = \\ \vec{H} \frac{\partial \vec{B}}{\partial t} + \vec{E} \frac{\partial \vec{D}}{\partial t} - \text{div}(\vec{H} \wedge \vec{E}) \end{array} \right. \\
 & \left\{ \begin{array}{l} c_i = \sqrt{\frac{nk_2}{nm}} = \sqrt{\frac{k_2}{m}} \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} c = \sqrt{\frac{1}{\epsilon_0 \mu_0}} \end{array} \right.
 \end{aligned}$$

$$\left\{ \vec{F}_{PK} = 2Q_\lambda \left(\frac{\vec{m}}{2} + \vec{v} \wedge n\vec{p} \right) \right. \Leftrightarrow \left. \left\{ \vec{F} = q(\vec{E} + \vec{v} \wedge \vec{B}) \right. \right.$$

Conclusion



The numerous analogies which appear between the eulerian theory of deformable media and the theories of electromagnetism, gravitation, special relativity, general relativity and even standard model of elementary particles, reinforced by the absence of particles analogue to magnetic monopoles, by a possible solution of the famous paradox of electron field energy and by the existence of a small asymmetry between curvature charges of vacancy or interstitial type, are sufficiently surprising and remarkable to alert any open and curious scientific spirit!

But it is also clear that these analogies are, by far, not perfect. It is then tantalizing to analyze much more carefully these analogies and to try to find how to perfect them.

