Eulerian theory of deformable media

Gérard Gremaud

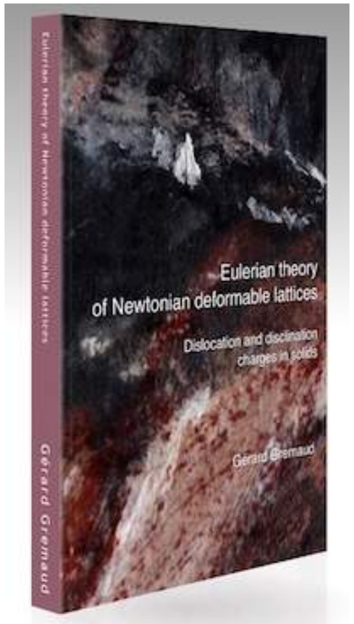
2016

Jenny

Contents

Eulerian theory of deformable media

Presses Polytechniques et Universitaires Romandes (PPUR), Lausanne, 2013, 750 pages (ISBN 978-2-88074-964-4)



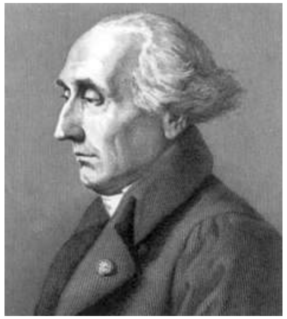
Amazon Press, Charleston (USA), 2016, 312 pages (ISBN 978-2-8399-1943-2)

http://gerardgremaud.ch

I - Eulerian theory of newtonian deformable medias

Coordinates systems

Differential geometries (Riemann-Cartan, Finsler, Kawaguchi, ...)

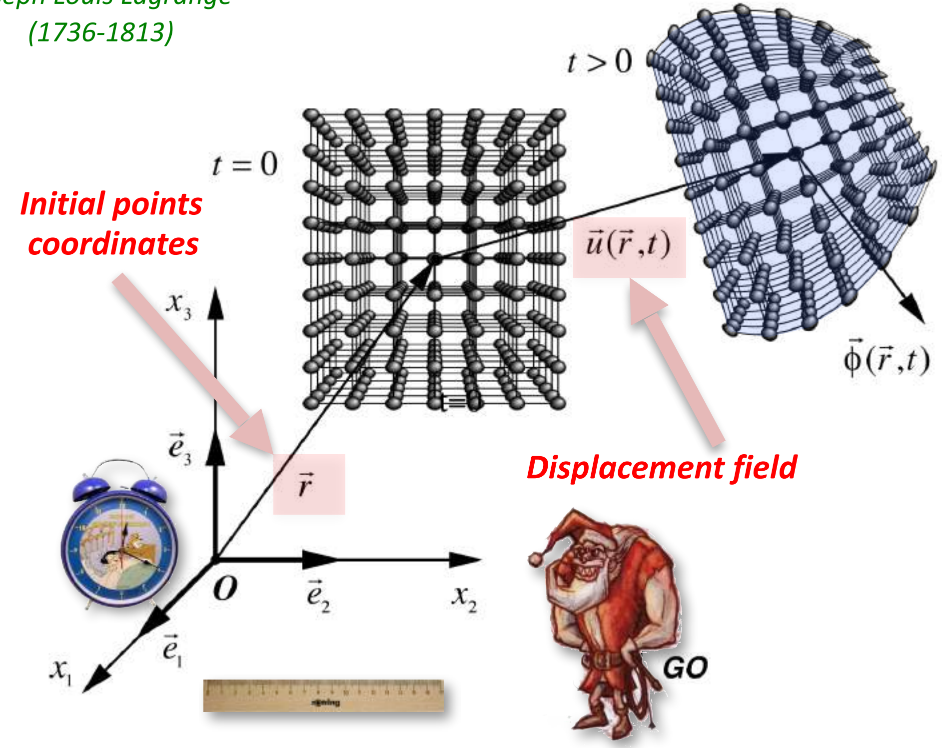


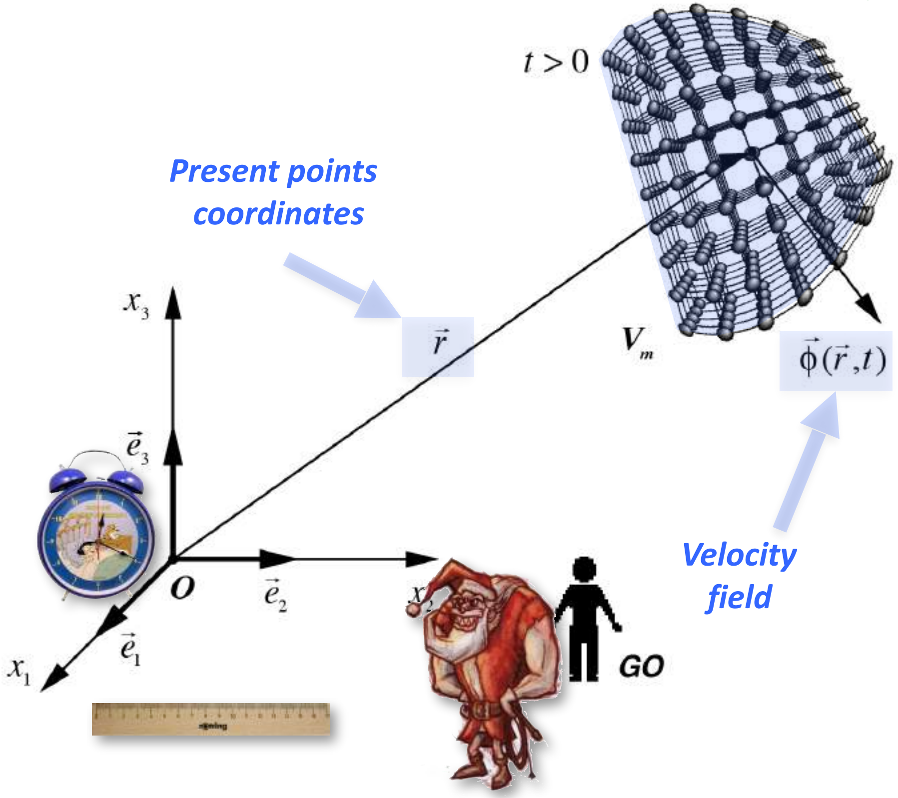
Joseph Louis Lagrange (1736-1813)

Lagrange coordinates

Euler coordinates

Leonhard Euler (1707-1783)

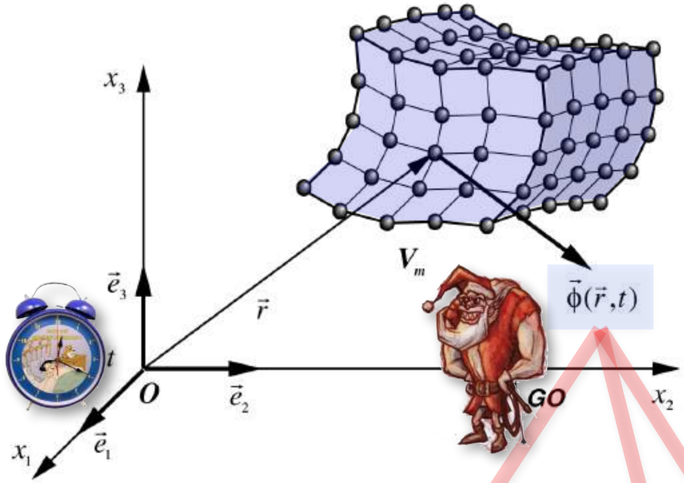




Geometrokinetic equations and distortion tensors in Euler coordinates

Temporal variations of the lattice « distortions »
are linked to the spatial variations of the velocity field

Vectorial representation of the distortion tensors



Material derivative

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\vec\phi\vec\nabla)$$

Tensor of distortion

$$\frac{d\vec\beta_i}{dt} = -\frac{S_n}{3n}\vec e_i + \overrightarrow{\mathrm{grad}}\,\phi_i$$

trace

Scalar of expansion

$$\frac{d\tau}{dt} = -\frac{S_n}{n} + \mathrm{div}\,\vec\phi$$

Antisymmetric part

Vector of rotation

$$\frac{d\vec\omega}{dt} = \frac{1}{2}\overrightarrow{\mathrm{rot}}\,\vec\phi$$

Distortion tensor $\vec\beta_i$

symmetry

$$\vec\varepsilon_i = \vec\beta_i - \vec e_i \wedge \vec\omega$$

antisymmetry

$$\vec\omega = -\frac{1}{2}\sum_k \vec e_k \wedge \vec\beta_k$$

trace

Strain tensor $\vec\varepsilon_i$

Rotation vector $\vec\omega$

without trace

trace

$$\vec\alpha_i = \vec\varepsilon_i - \frac{1}{3}\tau\vec e_i$$

$$\tau = \sum_k \vec e_k\vec\beta_k = \sum_k \vec e_k\vec\varepsilon_k$$

Shear strain tensor $\vec\alpha_i$

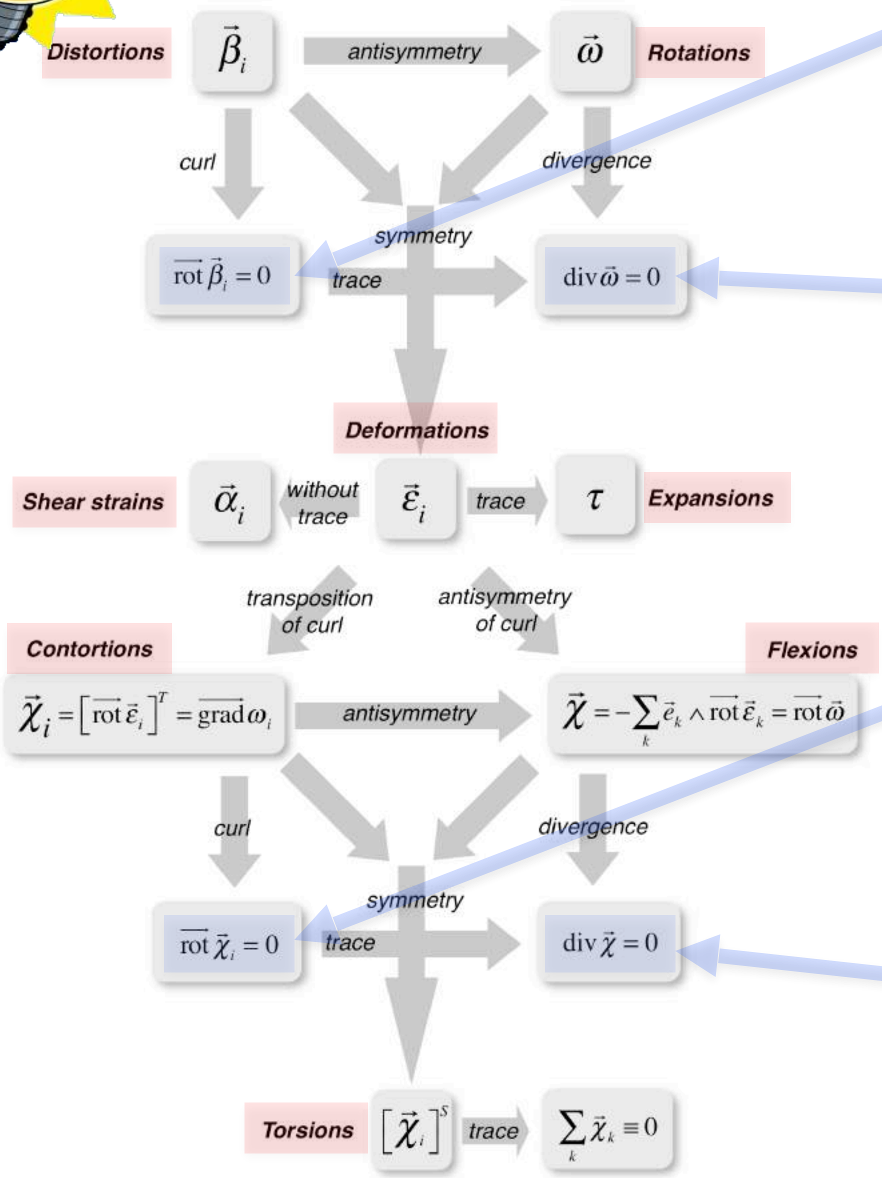
Volumic expansion scalar τ

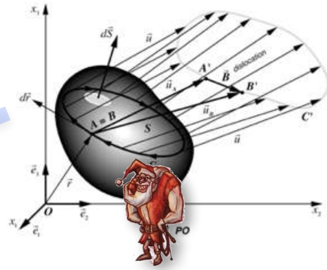
Relation with the volumic density n of sites and the volume v per site of the lattice

$$\tau = -\ln\frac{n}{n_0} = \ln\frac{v}{v_0}$$

Geometrocompatibility equations and contortion tensors in Euler coordinates

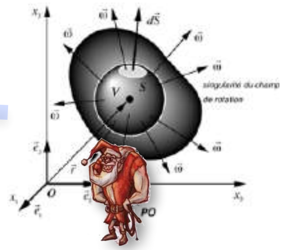
Distortion and contortion tensors and geometrocompatibility equations





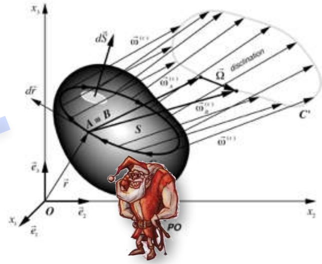
Continuity of the displacement field

$$\oint_C d\vec{u} = -\sum_k \vec{e}_k \iint_S \overrightarrow{\mathrm{rot}}\,\vec{\beta}_k d\vec{S} = 0 \quad ; \quad \forall \boldsymbol{C}$$



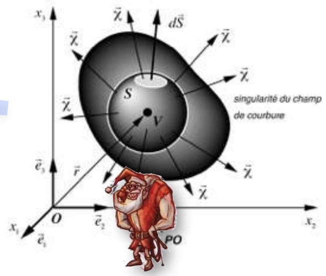
No singularity by divergence of the rotation field

$$\oiint_S \omega_\perp dS = \oiint_S \vec{\omega}\, d\vec{S} = \iiint_V \mathrm{div}\,\vec{\omega}\, dV = 0 \quad ; \quad \forall \boldsymbol{S}$$



Continuity of the rotation field associated to the deformations

$$\oint_C d\vec{\omega}^{(\varepsilon)} = \sum_k \vec{e}_k \iint_S \overrightarrow{\mathrm{rot}}\,\vec{\chi}_k\, d\vec{S} = \sum_k \vec{e}_k \iint_S \overrightarrow{\mathrm{rot}}\left[\overrightarrow{\mathrm{rot}}\,\vec{\varepsilon}_k\right]^{\mathrm{T}} d\vec{S} = 0$$



No singularity by divergence of the flexion field

$$\oiint_S \chi_\perp dS = \oiint_S \vec{\chi}\, d\vec{S} = \iiint_V \mathrm{div}\,\vec{\chi}\, dV = \sum_k \vec{e}_k \iiint_V \overrightarrow{\mathrm{rot}}\,\overrightarrow{\mathrm{rot}}\,\vec{\varepsilon}_k\, dV = 0$$

The only three necessary physical principles in Euler coordinates

Isaac Newton
(1643-1727)

Axiom of newtonian dynamics

$$e_{cin} = \frac{1}{2} m \vec{\phi}^2$$

Continuity principle for the newtonian inertial mass

$$\boxed{\frac{\partial \rho}{\partial t} = S_m - \operatorname{div}\left(\rho \vec{\phi} + \vec{J}_m\right) = S_m - \operatorname{div}\left(n \vec{p}\right) \quad (1)}$$

Axiom of the first principle of thermodynamics + **kinetic energy**

$$dU = \delta W + \delta Q \qquad e_{cin} = \frac{1}{2} m \vec{\phi}^2$$

Continuity principle for the total energy

$$\boxed{n\frac{du}{dt} + n\frac{de_{cin}}{dt} = S_w^{ext} - \operatorname{div}\vec{J}_w - \operatorname{div}\vec{J}_q - uS_n - e_{cin}S_n \quad (2)}$$

Sadi Carnot
(1837-1894)

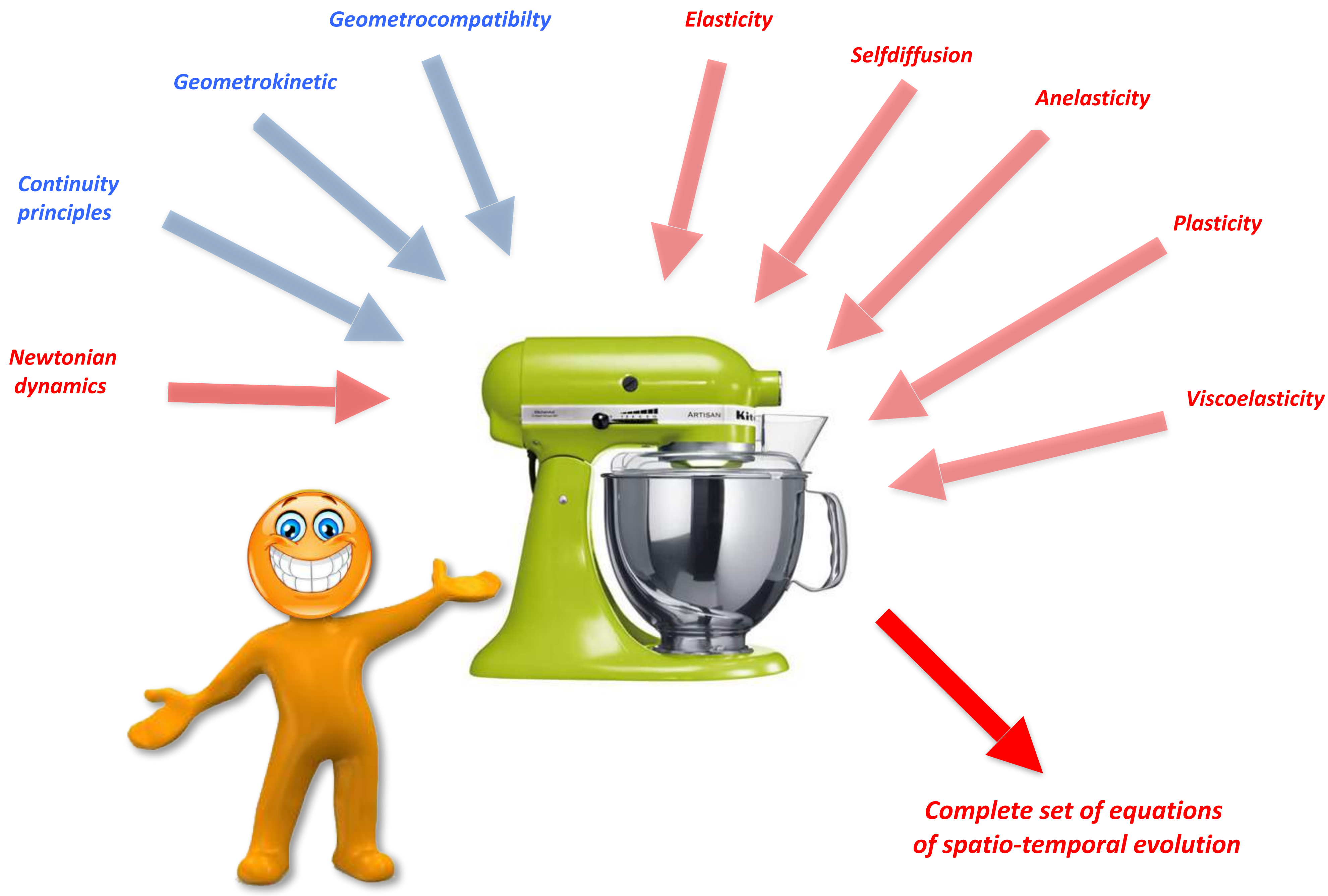
Axiom of the second principle of thermodynamics

$$dS \geq \frac{\delta Q}{T}$$

Continuity principle for the entropy

$$\boxed{n\frac{ds}{dt} = S_e - \operatorname{div}\left(\frac{\vec{J}_q}{T}\right) - s\,S_n \quad (3)}$$

Mix all these ingredients



The complete set of equations of spatio-temporal evolution in Euler coordinates

Fundamental equations

Topological equations

Topology

Equations topologiques

$$\begin{cases} \dfrac{d\vec{\beta}_i}{dt} = \overrightarrow{\mathrm{grad}}\,\phi_i & (1) \\ \dfrac{d\vec{\omega}}{dt} = \dfrac{1}{2}\overrightarrow{\mathrm{rot}}\,\vec{\phi} & (2) \\ \dfrac{d\tau}{dt} = \mathrm{div}\,\vec{\phi} & (3) \end{cases}$$

$$\begin{cases} \overrightarrow{\mathrm{rot}}\,\vec{\beta}_i = 0 & (4) \\ \mathrm{div}\,\vec{\omega} = 0 & (5) \\ d/dt = \partial/\partial t + (\vec{\varphi}\vec{\nabla}) & (6) \\ \vec{\varphi} = \vec{\phi} - \vec{\phi}_O(t) - \dot{\vec{\omega}}_O(t) \wedge \vec{r} & (7) \end{cases}$$

$$\begin{cases} \vec{\beta}_i = \vec{\beta}_i^{(\delta)} + \vec{e}_i \wedge \vec{\omega}_O(t) = \vec{\beta}_i^{\acute{e}l} + \vec{\beta}_i^{an} + \vec{\beta}_i^{pl} + \vec{e}_i \wedge \vec{\omega}_O(t) & (8) \\ \vec{\omega} = -\dfrac{1}{2}\displaystyle\sum_k \vec{e}_k \wedge \vec{\beta}_k = \vec{\omega}^{(\delta)} + \vec{\omega}_0(t) = \vec{\omega}^{\acute{e}l} + \vec{\omega}^{an} + \vec{\omega}^{pl} + \vec{\omega}_0(t) & (9) \\ \tau = \displaystyle\sum_k \vec{\beta}_k \vec{e}_k = \sum_k \vec{\beta}_k^{(\delta)} \vec{e}_k = \tau^{\acute{e}l} + \tau^{pl} \qquad (\tau^{an} \equiv 0 \ \textit{par hypothèse}) & (10) \\ \vec{\varepsilon}_i = \vec{\beta}_i - \vec{e}_i \wedge \vec{\omega} = \vec{\beta}_i^{(\delta)} - \vec{e}_i \wedge \vec{\omega}^{(\delta)} = \vec{\varepsilon}_i^{\acute{e}l} + \vec{\varepsilon}_i^{an} + \vec{\varepsilon}_i^{pl} & (11) \\ \vec{\alpha}_i = \vec{\varepsilon}_i - \dfrac{1}{3}\tau\vec{e}_i = \vec{\alpha}_i^{\acute{e}l} + \vec{\alpha}_i^{an} + \vec{\alpha}_i^{pl} & (12) \end{cases}$$

Dynamics

Equations dynamique

$$n\frac{d\vec{p}}{dt} = \rho\vec{g} + \sum_k \vec{e}_k\,\mathrm{div}\,\vec{s}_k - \frac{1}{2}\overrightarrow{\mathrm{rot}}\,\vec{m} - \overrightarrow{\mathrm{grad}}\,p + nm\vec{\phi}_I\frac{dC_I}{dt} - nm\vec{\phi}_L\frac{dC_L}{dt} \qquad (13)$$

$$\begin{cases} n = 1/v = n_0\exp(-\tau^{\acute{e}l}) & (14) \\ \vec{p} = m\left(\vec{\phi} + C_I\vec{\phi}_I - C_L\vec{\phi}_L\right) = m\vec{\phi} + m\left(C_I - C_L\right)\vec{\phi} + \dfrac{m}{n}\left(\vec{J}_I - \vec{J}_L\right) \\ \qquad = \left[\rho\vec{\phi} + m\left(\vec{J}_I - \vec{J}_L\right)\right]/n & (15) \\ \rho = mn\left(1 + C_I - C_L\right) & (16) \end{cases}$$

Diffusion

Equations de diffusion

$$\begin{cases} n\dfrac{dC_L}{dt} = \left(S_{I-L} + S_L^{pl}\right) - C_L\left(S_L^{pl} - S_I^{pl}\right) - \mathrm{div}\,\vec{J}_L & (17) \\ n\dfrac{dC_I}{dt} = \left(S_{I-L} + S_I^{pl}\right) - C_I\left(S_L^{pl} - S_I^{pl}\right) - \mathrm{div}\,\vec{J}_I & (18) \end{cases}$$

$$\begin{cases} \vec{J}_L = nC_L\Delta\vec{\varphi}_L = nC_L\left(\vec{\phi}_L - \vec{\phi}\right) = nC_L\left(\vec{\varphi}_L - \vec{\varphi}\right) & (19) \\ \vec{J}_I = nC_I\Delta\vec{\varphi}_I = nC_I\left(\vec{\phi}_I - \vec{\phi}\right) = nC_I\left(\vec{\varphi}_I - \vec{\varphi}\right) & (20) \end{cases}$$

Heat equation

Thermic

Equations thermiques

$$\begin{aligned} nT\frac{ds}{dt} = {}& -\left(\mu_L^* + \mu_I^*\right)S_{I-L} - \left(\mu_L^* + h^*\right)S_L^{pl} - \left(\mu_I^* - h^*\right)S_I^{pl} + T\vec{J}_L\vec{X}_L \\ & + T\vec{J}_I\vec{X}_I + \vec{s}_k^{\,dis}\frac{d\vec{\beta}_k^{an}}{dt} + \vec{m}^{dis}\frac{d\vec{\omega}^{an}}{dt} + \vec{s}_k\frac{d\vec{\beta}_k^{pl}}{dt} + \vec{m}\frac{d\vec{\omega}^{pl}}{dt} - \mathrm{div}\,\vec{J}_q \end{aligned} \qquad (21)$$

$$\begin{cases} \mu_L^* = \mu_L - \dfrac{1}{2}m\left(\vec{\phi}_L^{\,2} - 2\Delta\vec{\varphi}_L^2\right) & (22) \\ \mu_I^* = \mu_I + \dfrac{1}{2}m\vec{\phi}_I^{\,2} & (23) \end{cases}$$

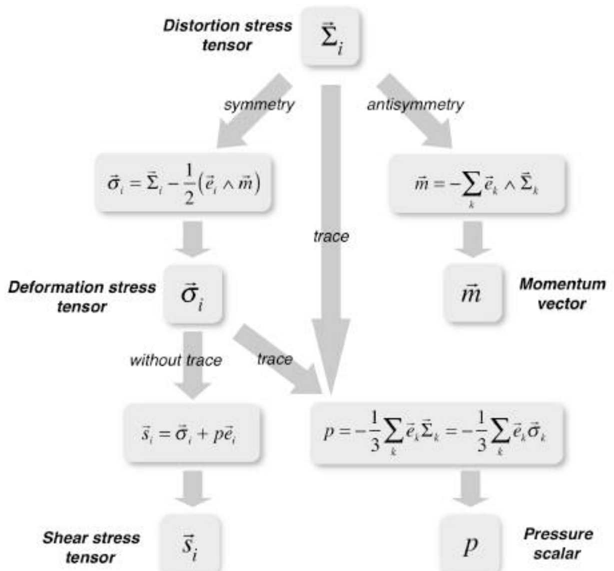
$$\begin{cases} \vec{X}_q = \overrightarrow{\mathrm{grad}}\,\dfrac{1}{T} & (24) \\ \vec{X}_L = \dfrac{1}{T}\left(-\overrightarrow{\mathrm{grad}}\ \mu_L^* + m\dfrac{d}{dt}\left(\vec{\phi}_L - 2\Delta\vec{\varphi}_L\right) - m\vec{g}\right) & (25) \\ \vec{X}_I = \dfrac{1}{T}\left(-\overrightarrow{\mathrm{grad}}\ \mu_I^* - m\dfrac{d}{dt}\left(\vec{\phi}_I\right) + m\vec{g}\right) & (26) \end{cases}$$

$$h^* = f + Ts + pv + \frac{1}{2}m\vec{\phi}^2 - \mu_L C_L - \mu_I C_I \qquad (27)$$

Newton equation

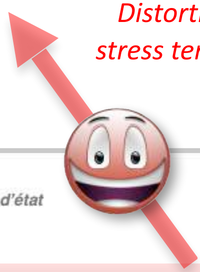
Selfdiffusion equations

The complete set of equations of spatio-temporal evolution in Euler coordinates



Phenomenological equations :
state equations and dissipative equations

Distortion
stress tensors



State function and equations

Fonctions et équations d'état

$$f = f\left(\alpha_{ij}^{\acute{e}l}, \alpha_{ij}^{an}, \omega_k^{\acute{e}l}, \omega_k^{an}, \tau^{\acute{e}l}, C_L, C_I, T\right) \qquad (28)$$

$$\begin{cases} s_{ij} = \dfrac{n}{2}\left(\dfrac{\partial f}{\partial \alpha_{ij}^{\acute{e}l}} + \dfrac{\partial f}{\partial \alpha_{ji}^{\acute{e}l}}\right) - \dfrac{n}{3}\delta_{ij}\sum_k \dfrac{\partial f}{\partial \alpha_{kk}^{\acute{e}l}} = s_{ij}(\alpha_{lm}^{\acute{e}l}, \omega_n^{\acute{e}l}, \tau^{\acute{e}l}, \alpha_{lm}^{an}, \omega_n^{an}, C_L, C_I, T) & (29) \\ m_k = n\dfrac{\partial f}{\partial \omega_k^{\acute{e}l}} = m_k(\alpha_{lm}^{\acute{e}l}, \omega_n^{\acute{e}l}, \tau^{\acute{e}l}, \alpha_{lm}^{an}, \omega_n^{an}, C_L, C_I, T) & (30) \\ p = -n\dfrac{\partial f}{\partial \tau^{\acute{e}l}} = p(\alpha_{lm}^{\acute{e}l}, \omega_n^{\acute{e}l}, \tau^{\acute{e}l}, \alpha_{lm}^{an}, \omega_n^{an}, C_L, C_I, T) & (31) \end{cases}$$

Elasticity

$$\begin{cases} s_{ij}^{dis} = \dfrac{n}{2}\left(\dfrac{\partial f}{\partial \alpha_{ij}^{an}} + \dfrac{\partial f}{\partial \alpha_{ji}^{an}}\right) - \dfrac{n}{3}\delta_{ij}\sum_k \dfrac{\partial f}{\partial \alpha_{kk}^{an}} = s_{ij}^{dis}(\alpha_{lm}^{\acute{e}l}, \omega_n^{\acute{e}l}, \tau^{\acute{e}l}, \alpha_{lm}^{an}, \omega_n^{an}, C_L, C_I, T) & (32) \\ m_k^{dis} = n\dfrac{\partial f}{\partial \omega_k^{an}} = m_k^{dis}(\alpha_{lm}^{\acute{e}l}, \omega_n^{\acute{e}l}, \tau^{\acute{e}l}, \alpha_{lm}^{an}, \omega_n^{an}, C_L, C_I, T) & (33) \end{cases}$$

Anelasticity

State equations

$$\begin{cases} s = -\dfrac{\partial f}{\partial T} = s(\alpha_{lm}^{\acute{e}l}, \omega_n^{\acute{e}l}, \tau^{\acute{e}l}, \alpha_{lm}^{an}, \omega_n^{an}, C_L, C_I, T) & (34) \\ \mu_L = \dfrac{\partial f}{\partial C_L} = \mu_L(\alpha_{lm}^{\acute{e}l}, \omega_n^{\acute{e}l}, \tau^{\acute{e}l}, \alpha_{lm}^{an}, \omega_n^{an}, C_L, C_I, T) & (35) \\ \mu_I = \dfrac{\partial f}{\partial C_I} = \mu_I(\alpha_{lm}^{\acute{e}l}, \omega_n^{\acute{e}l}, \tau^{\acute{e}l}, \alpha_{lm}^{an}, \omega_n^{an}, C_L, C_I, T) & (36) \end{cases}$$

Thermicity

Selfdiffusivity

Selfdiffusion

Equations de dissipation: auto-diffusion et création-annihilation de paires

$$\begin{cases} \vec{J}_q = \vec{J}_q(\vec{X}_q, \vec{X}_L, \vec{X}_I, n, T, C_L, C_I, ...) & (37) \\ \vec{J}_L = \vec{J}_L(\vec{X}_q, \vec{X}_L, \vec{X}_I, n, T, C_L, C_I, ...) & (38) \\ \vec{J}_I = \vec{J}_I(\vec{X}_q, \vec{X}_L, \vec{X}_I, n, T, C_L, C_I, ...) & (39) \end{cases}$$

$$S_{I-L} = S_{I-L}\left(\mu_L^* + \mu_I^*, n, T, C_L, C_I, ...\right) \qquad (40)$$

+ Creation-annihilation of point defect pairs

Anelasticity

Equations de dissipation: anélasticité

$$\begin{cases} \vec{s}_i = \vec{s}_i^{cons}\left(\vec{\alpha}_m^{an}, v, T, ...\right) + \vec{s}_i^{dis}\left(\dfrac{d\vec{\alpha}_m^{an}}{dt}, v, T, ...\right) & (41) \\ \vec{m} = \vec{m}^{cons}\left(\vec{\omega}^{an}, v, T, ...\right) + \vec{m}^{dis}\left(\dfrac{d\vec{\omega}^{an}}{dt}, v, T, ...\right) & (42) \end{cases}$$

Plasticity

Equations de dissipation: plasticité

$$\begin{cases} \dfrac{d\tau^{pl}}{dt} = \dfrac{S_n}{n} = \dfrac{1}{n}\left(S_L^{pl} - S_I^{pl}\right) & (43) \\ S_L^{pl} = S_L^{pl}\left[\left(\mu_L^* + g^*\right), v, T, C_L, C_I, ...\right] & (44) \\ S_I^{pl} = S_I^{pl}\left[\left(\mu_I^* - g^*\right), v, T, C_L, C_I, ...\right] & (45) \end{cases}$$

$$g^* = f + pv + m\vec{\phi}^2/2 - \mu_L C_L - \mu_I C_I \qquad (46)$$

$$\begin{cases} \dfrac{d\vec{\alpha}_i^{pl}}{dt} = \dfrac{d\vec{\alpha}_i^{pl}}{dt}(\vec{s}_m, v, T, ...) & (47) \\ \dfrac{d\vec{\omega}^{pl}}{dt} = \dfrac{d\vec{\omega}^{pl}}{dt}(\vec{m}, v, T, ...) & (48) \end{cases}$$

+ Creation-annihilation of point defects

The complete set of equations of spatio-temporal evolution in Euler coordinates

Additional equations

Continuité de la masse

Inertial mass continuity

$$\frac{\partial \rho}{\partial t} = -\mathrm{div}\left[\rho\vec{\phi} + m\left(\vec{J}_I - \vec{J}_L\right)\right] = -\mathrm{div}(n\vec{p}) \qquad \text{dans } Q\xi_1\xi_2\xi_3 \tag{49}$$

Flux de travail et force de surface

Work flux

Surface forces

$$\begin{cases} \vec{J}_w = \mu_L^{\ *}\vec{J}_L + \mu_I^{\ *}\vec{J}_I - \phi_k\vec{s}_k - \dfrac{1}{2}\left(\vec{\phi}\wedge\vec{m}\right) + p\vec{\phi} & (50) \\ \vec{F}_S = \displaystyle\sum_k \vec{e}_k\left(\vec{s}_k\vec{\boldsymbol{n}}\right) + \dfrac{1}{2}\left(\vec{m}\wedge\vec{\boldsymbol{n}}\right) - \vec{\boldsymbol{n}}p & (51) \end{cases}$$

Source d'entropie

Entropy source

$$\begin{aligned} S_e = &-\frac{1}{T}\left(\mu_L^* + \mu_I^*\right)S_{I-L} - \frac{1}{T}\left(\mu_L^* + g^*\right)S_L^{pl} - \frac{1}{T}\left(\mu_I^* - g^*\right)S_I^{pl} \\ &+\vec{J}_L\vec{X}_L + \vec{J}_I\vec{X}_I + \frac{1}{T}\left(\vec{s}_k^{\,dis}\frac{d\vec{\beta}_k^{an}}{dt} + \vec{m}^{dis}\frac{d\vec{\omega}^{an}}{dt} + \vec{s}_k\frac{d\vec{\beta}_k^{pl}}{dt} + \vec{m}\frac{d\vec{\omega}^{pl}}{dt}\right) \\ &+\vec{J}_q\,\overrightarrow{\mathrm{grad}}\left(\frac{1}{T}\right) \end{aligned} \tag{52}$$

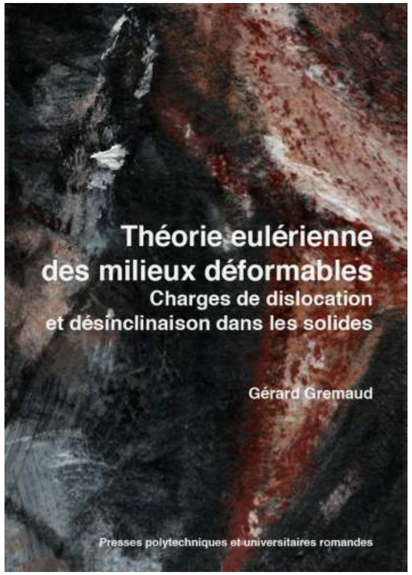
Bilan énergétique

Energy balance

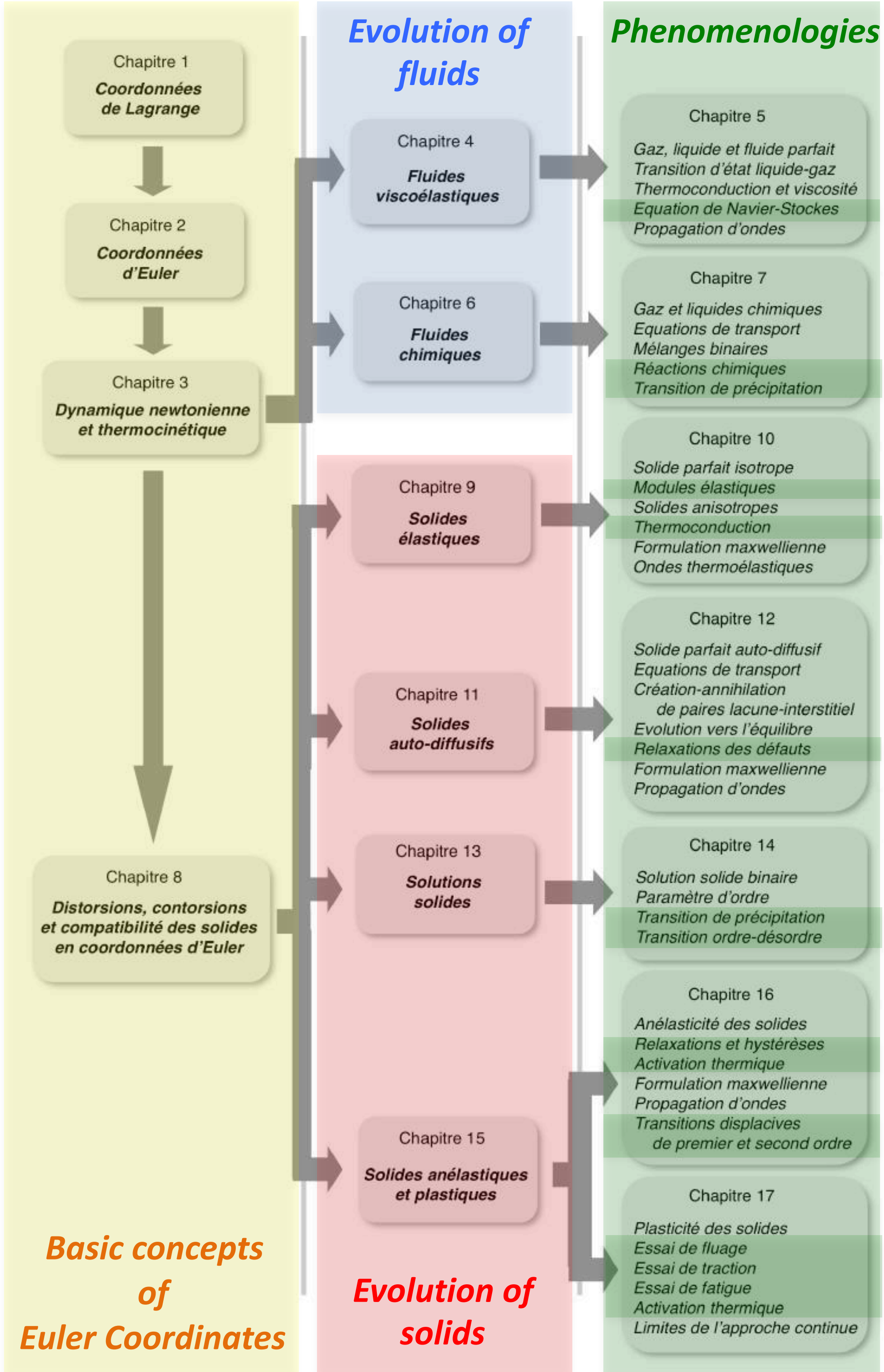
$$\begin{aligned} &n\vec{\phi}\left(\frac{d\vec{p}}{dt} - m\vec{\phi}_I\frac{dC_I}{dt} + m\vec{\phi}_L\frac{dC_L}{dt}\right) + \vec{s}_k\frac{d\vec{\beta}_k}{dt} + \vec{m}\frac{d\vec{\omega}}{dt} - p\frac{d\tau}{dt} \\ &\qquad = \rho\vec{g}\vec{\phi} - \mathrm{div}\left[-\phi_k\vec{s}_k - \frac{1}{2}\left(\vec{\phi}\wedge\vec{m}\right) + p\vec{\phi}\right] \end{aligned} \tag{53}$$

Poynting vector

II - Application: phenomenologies of usual fluids and solids



first part



III – Dislocation and disclination charges

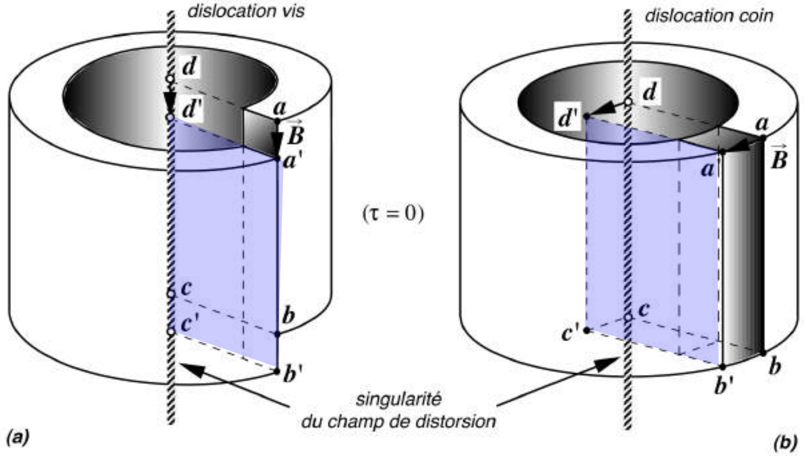
What's a line of topological singularity?

Vito Volterra
(1860-1940)

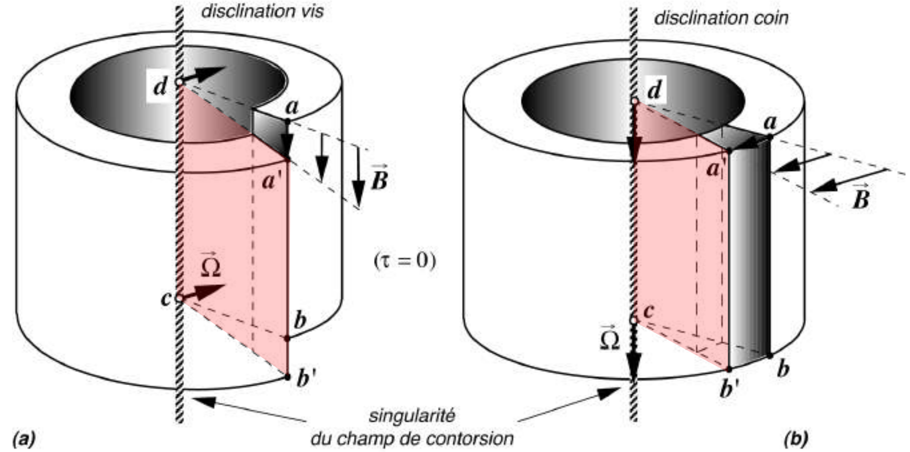
Singularity line by translation

Singularity line by rotation

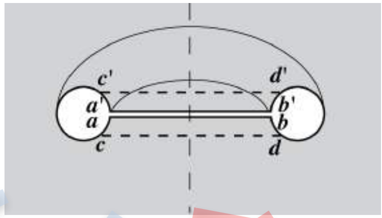
Screw dislocation — **Edge dislocation**



Twist disclination — **Wedge disclination**



What's a loop of topological singularity?

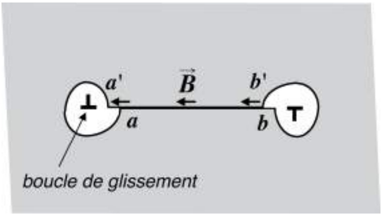


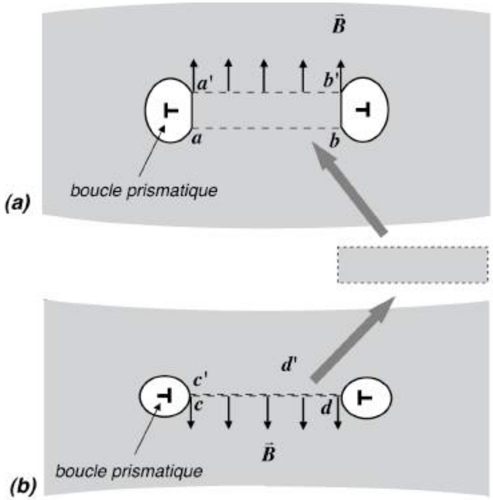
Mixed dislocation loop by translation

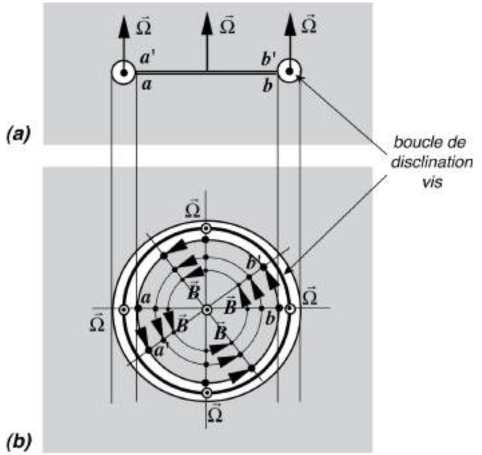
Edge dislocation loop by material addition or substraction

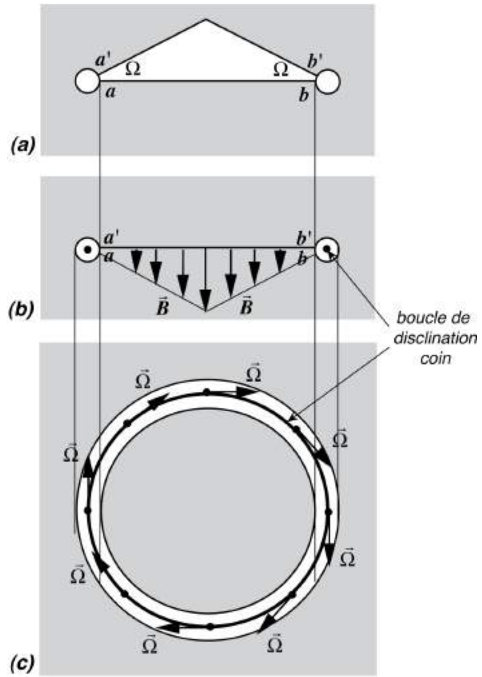
Twist disclination loop by rotation

Wedge disclination loop by material addition or substraction



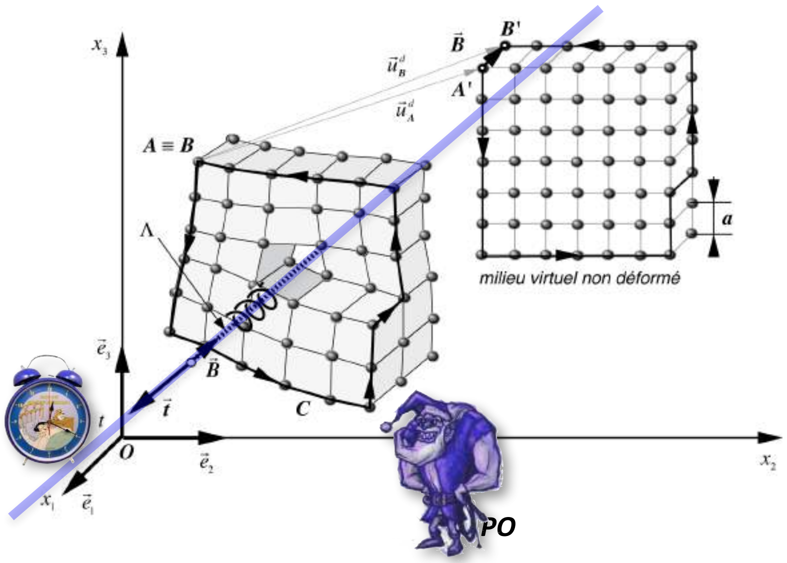




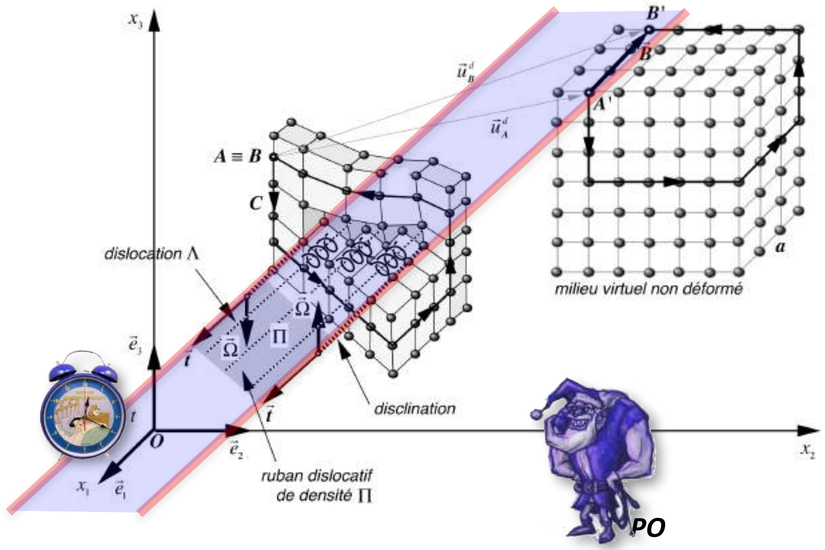


Quantification of the topological singularities as strings or membranes in solid lattices

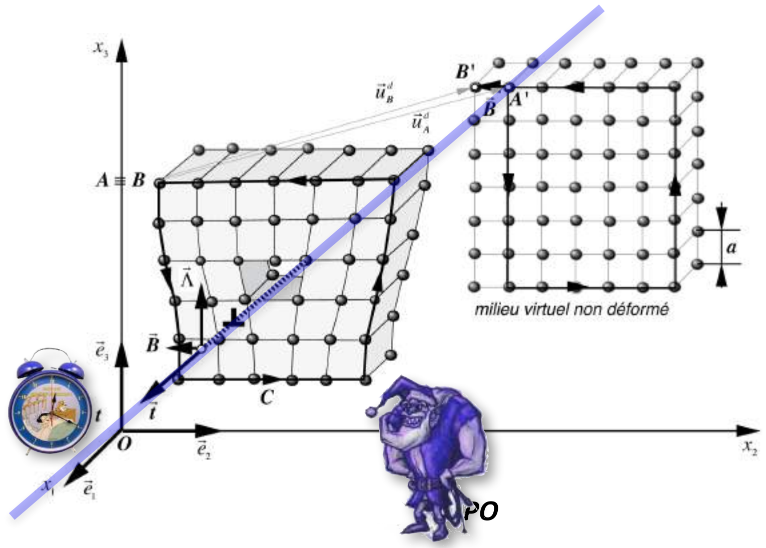
Screw dislocation string



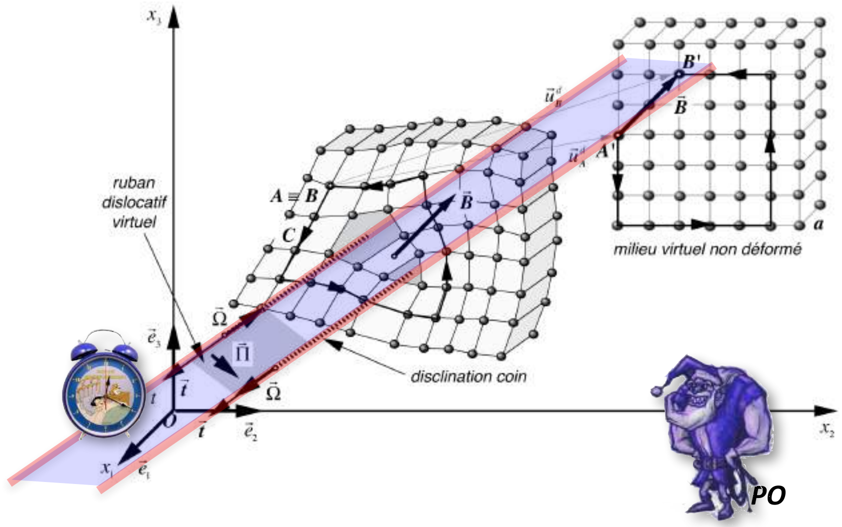
Srew dislocation membrane limited by two twist disclination strings



Edge dislocation string

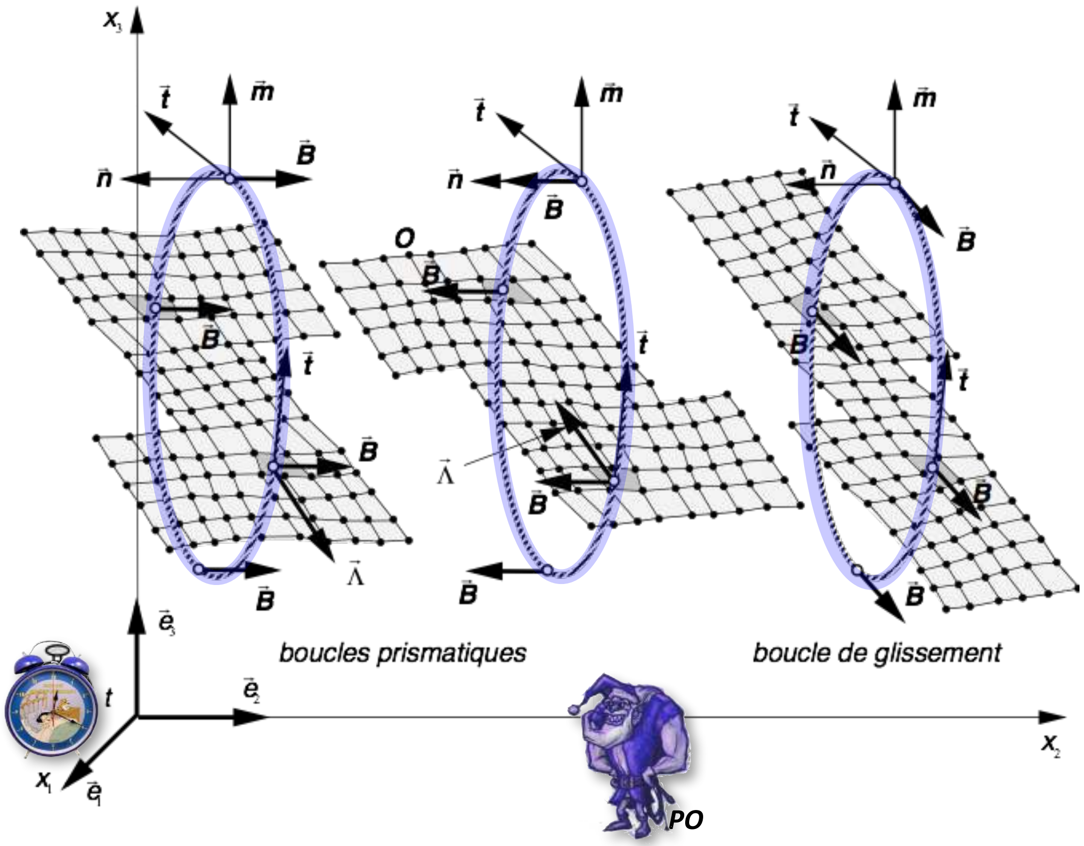


Edge dislocation membrane limited by two wedge disclination strings

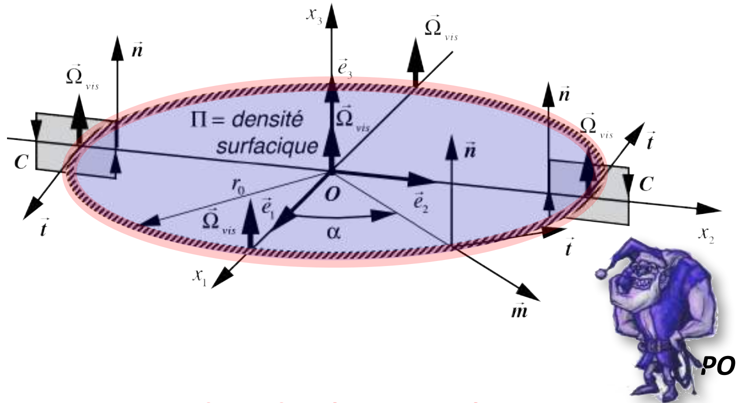


Quantification of the topological singularities as loops and membranes in solid lattices

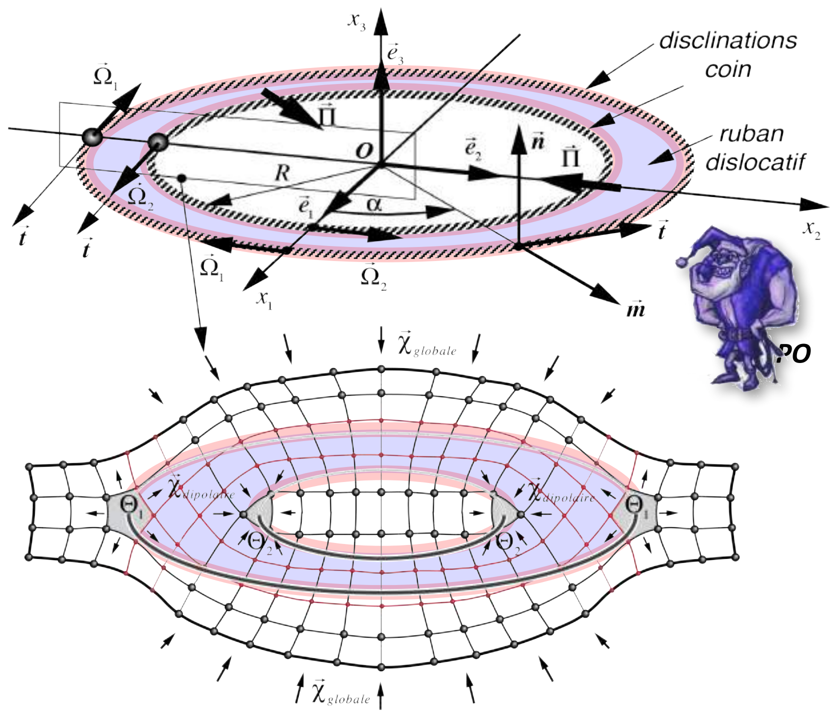
Edge and mixed dislocation loops



Twist disclination loop
with screw dislocation membrane



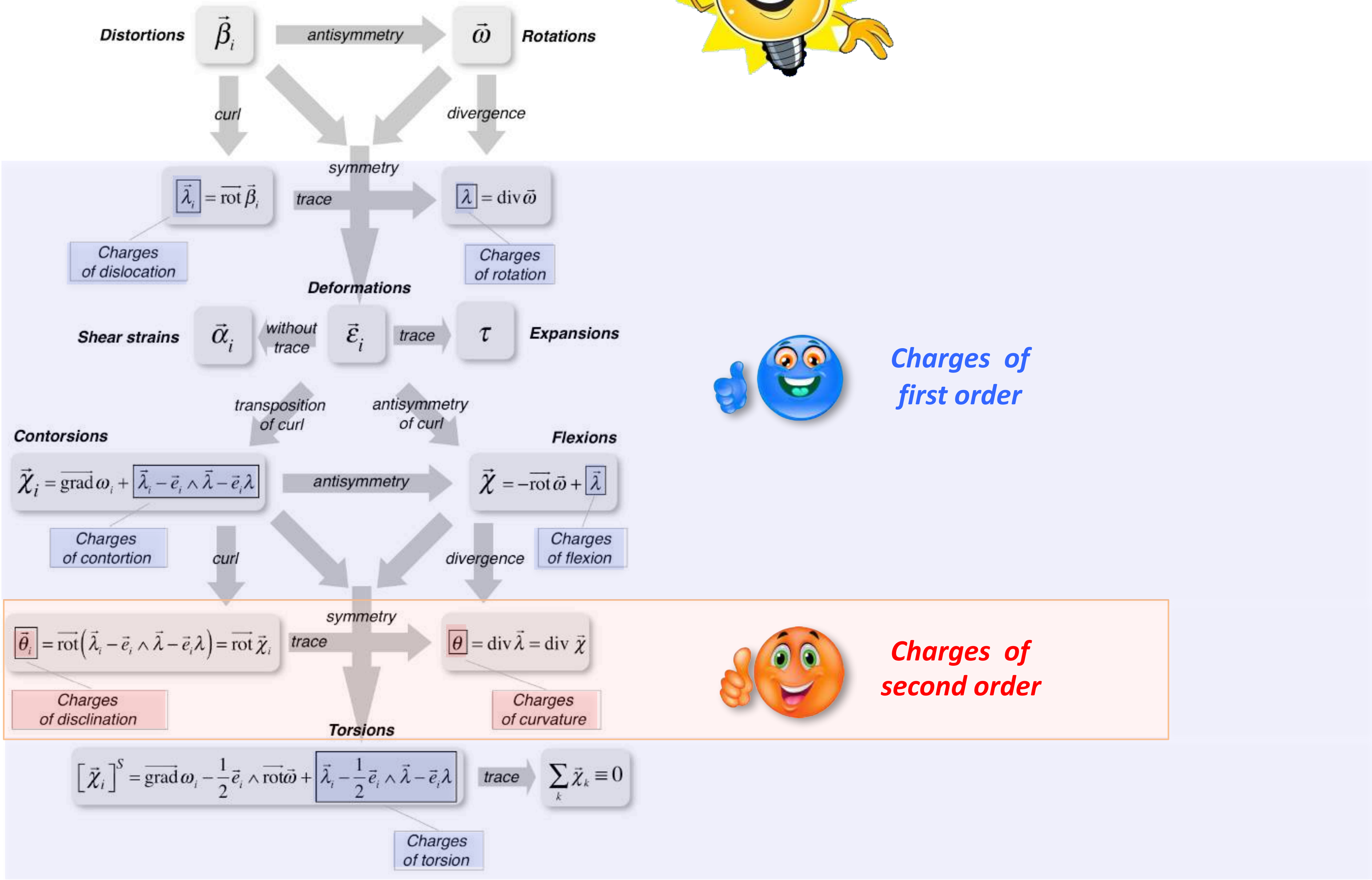
Wedge disclination loop
with edge dislocation membrane



Incompatibility charges

associated to the topological singularities (strings, membranes and loops) of a solid lattice

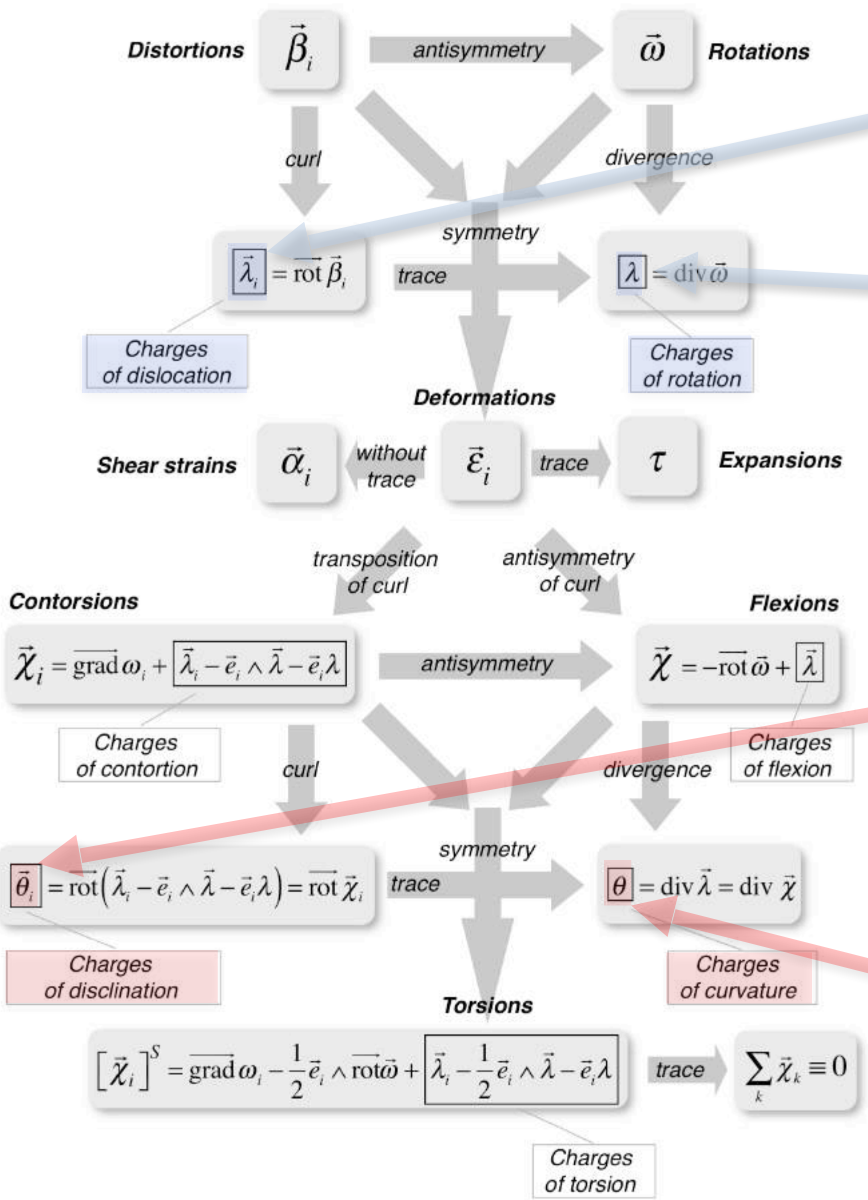
Incompatibility equations and charge tensors



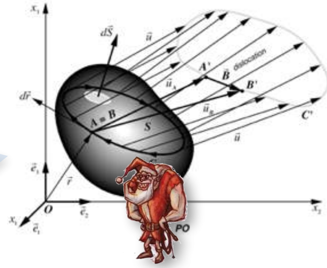
Incompatibility charges

associated to the topological singularities (strings, membranes and loops) of a solid lattice

Incompatibility equations and charge tensors



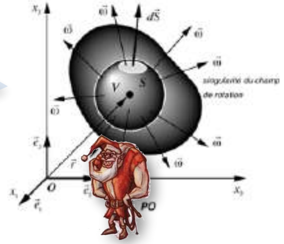
Discontinuity of the displacement field



Dislocation Burgers vector

$$\vec{B} = \vec{u}_B - \vec{u}_A = \oint_C \delta\vec{u} = -\sum_k \vec{e}_k \iint_S \overrightarrow{\mathrm{rot}}\,\vec{\beta}_k\, d\vec{S} = -\sum_k \vec{e}_k \iint_S \vec{\lambda}_k\, d\vec{S} \neq 0$$

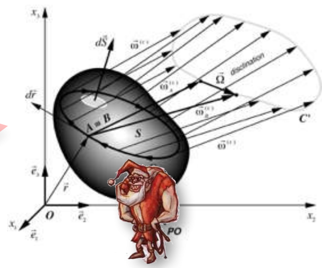
Singularity of the divergence of the rotation field



$$Q_\lambda = \oiint_S \vec{\omega}\, d\vec{S} = \iiint_V \mathrm{div}\,\vec{\omega}\, dV = \iiint_V \lambda\, dV \neq 0$$

Macroscopic scalar charge of rotation

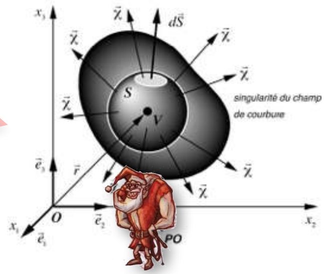
Discontinuity of the rotation field associated to the deformations



Disclination Franck vector

$$\vec{\Omega} = \oint_C \delta\vec{\omega}^{\varepsilon} = \sum_k \vec{e}_k \iint_S \overrightarrow{\mathrm{rot}}\,\vec{\chi}_k\, d\vec{S} = \sum_k \vec{e}_k \iint_S \vec{\theta}_i\, d\vec{S} = \sum_k \vec{e}_k \oint_C \left(\vec{\lambda}_k - \vec{e}_k \wedge \vec{\lambda} - \vec{e}_k \lambda\right) d\vec{r} \neq 0$$

Singularity of the divergence of the flexion field

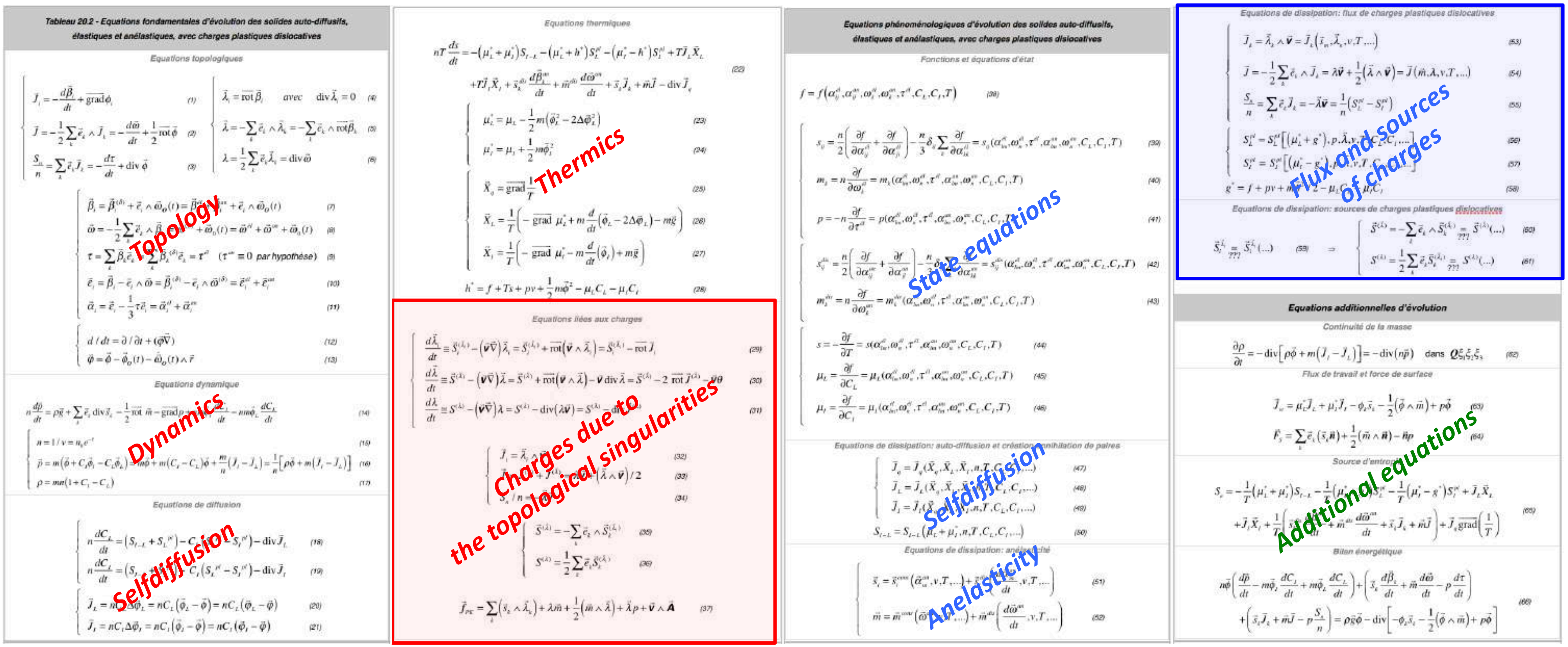


$$Q_\theta = \oiint_S \vec{\chi}\, d\vec{S} = \iiint_V \mathrm{div}\,\vec{\chi}\, dV = \iiint_V \theta\, dV \neq 0$$

Macroscopic scalar charge of flexion

The complete set of equations of spatio-temporal evolution of a charged lattice

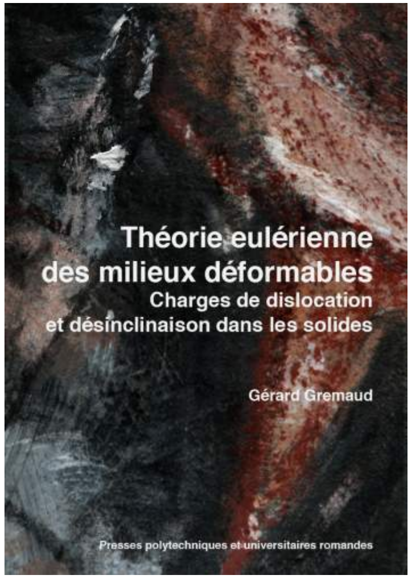
Fundamental equations **Phenomenological equations**

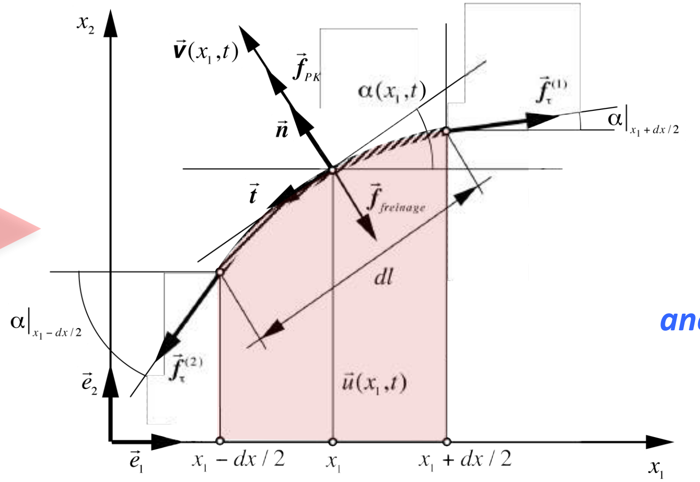


Additional equations

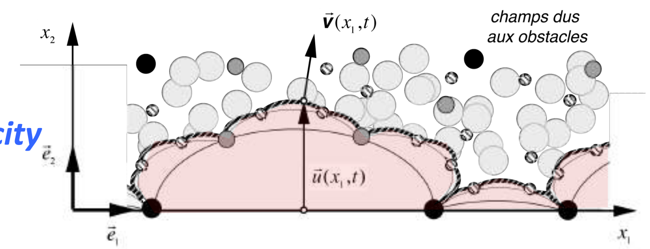
IV - Application: elements of dislocation theory in usual solids

String model of a dislocation line

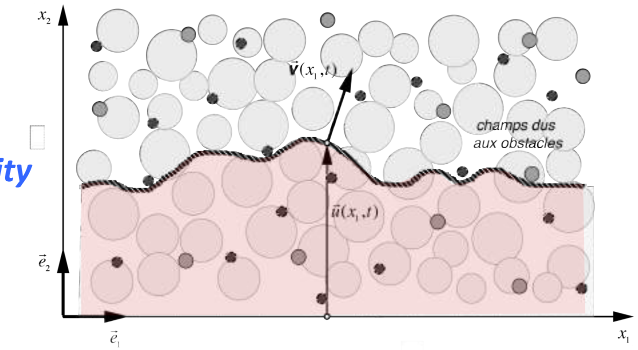




anelasticity



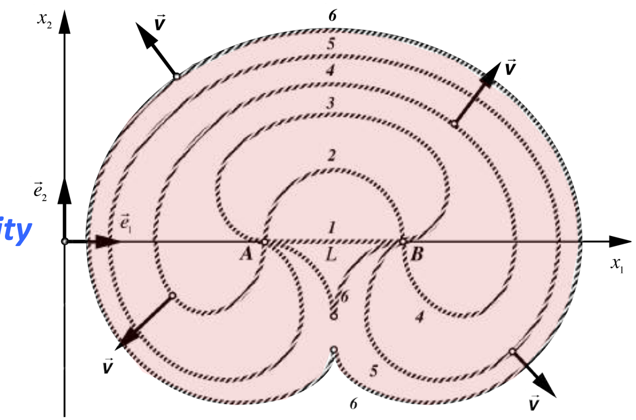
plasticity



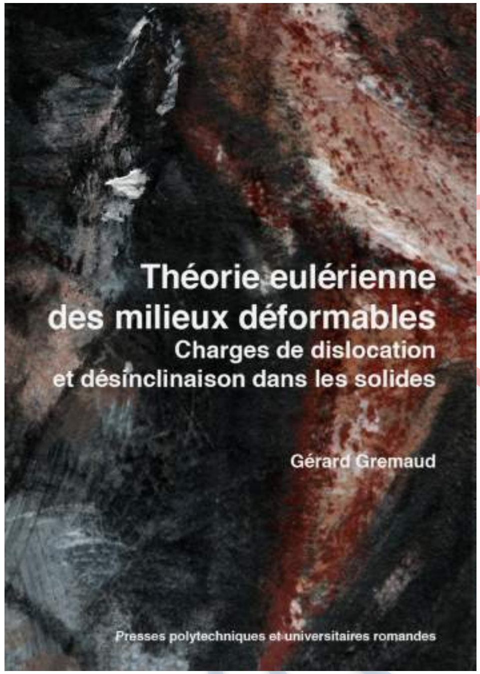
$$\left(M_0 \frac{\partial^2 u}{\partial t^2} + B_f \frac{\partial u}{\partial t}\right)\sqrt{1+(\partial u/\partial x_1)^2} - \frac{\partial}{\partial x_1}\left[\tau \frac{\partial u}{\partial x_1} \Big/ \sqrt{1+(\partial u/\partial x_1)^2}\right]$$

$$= \boldsymbol{B}s_{23} + \sum_{n=1}^{N} \boldsymbol{f}_n\left(x_1,u(x_1,t)\right) + \boldsymbol{F}_{fluctuation}\left(x_1,u(x_1,t),t\right)$$

plasticity



Other consequences



Relativistic dynamics of the charges

Interactions of electrical type and of gravitational type between charges

String model of the dislocation line

+

Absence of particles analogue to magnetic monopoles

Possible solution of the famous paradox of the electron field energy

Existence of a small asymmetry between curvature charges of vacancy or interstitial type

Maxwell equations and Lorentz force at constant volumic expansion

$$\begin{cases} -\dfrac{d(2\vec{\omega})}{dt} + \overrightarrow{\mathrm{rot}}\,\vec{\phi} = (2\vec{J}) \\ \mathrm{div}\,(2\vec{\omega}) = (2\lambda) \end{cases} \Leftrightarrow \begin{cases} -\dfrac{\partial \vec{D}}{\partial t} + \overrightarrow{\mathrm{rot}}\,\vec{H} = \vec{j} \\ \mathrm{div}\,\vec{D} = \rho \end{cases}$$

$$\begin{cases} \dfrac{d(n\vec{p})}{dt} = -\overrightarrow{\mathrm{rot}}\left(\dfrac{\vec{m}}{2}\right) \\ \mathrm{div}(n\vec{p}) = 0 \end{cases} \Leftrightarrow \begin{cases} \dfrac{\partial \vec{B}}{\partial t} = -\overrightarrow{\mathrm{rot}}\,\vec{E} \\ \mathrm{div}\,\vec{B} = 0 \end{cases}$$

$$\begin{cases} (2\vec{\omega}) = \left(\dfrac{1}{nk_2}\right)\left(\dfrac{\vec{m}}{2}\right) + (2\vec{\omega}^{an}) + (2\vec{\omega}_0(t)) \\ (n\vec{p}) = (nm)\left[\vec{\phi} + (C_I - C_L)\vec{\phi} + \left(\dfrac{1}{n}\left(\vec{J}_I - \vec{J}_L\right)\right)\right] \end{cases} \Leftrightarrow \begin{cases} \vec{D} = \varepsilon_0\,\vec{E} + \vec{P} + \vec{P}_0(t) \\ \vec{B} = \mu_0\left[\vec{H} + \left(\chi^{para} + \chi^{dia}\right)\vec{H} + \vec{M}\right] \end{cases}$$

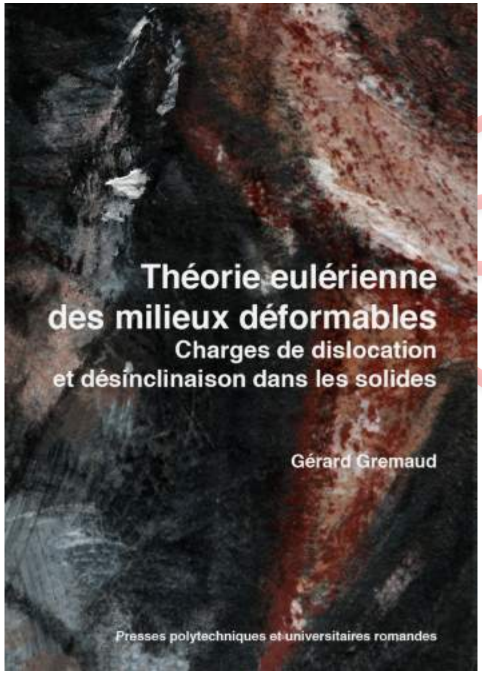
$$\begin{cases} \dfrac{d(2\lambda)}{dt} = -\mathrm{div}(2\vec{J}) \end{cases} \Leftrightarrow \begin{cases} \dfrac{\partial \rho}{\partial t} = -\mathrm{div}\,\vec{j} \end{cases}$$

$$\begin{cases} -\left(\dfrac{\vec{m}}{2}\right)(2\vec{J}) = \\ \quad \vec{\phi}\,\dfrac{d(n\vec{p})}{dt} + \left(\dfrac{\vec{m}}{2}\right)\dfrac{d(2\vec{\omega})}{dt} - \mathrm{div}\left(\vec{\phi} \wedge \left(\dfrac{\vec{m}}{2}\right)\right) \end{cases} \Leftrightarrow \begin{cases} -\vec{E}\,\vec{j} = \\ \quad \vec{H}\,\dfrac{\partial \vec{B}}{\partial t} + \vec{E}\,\dfrac{\partial \vec{D}}{\partial t} - \mathrm{div}\left(\vec{H} \wedge \vec{E}\right) \end{cases}$$

$$\begin{cases} c_t = \sqrt{\dfrac{nk_2}{nm}} = \sqrt{\dfrac{k_2}{m}} \end{cases} \Leftrightarrow \begin{cases} c = \sqrt{\dfrac{1}{\varepsilon_0 \mu_0}} \end{cases}$$

$$\begin{cases} \vec{\boldsymbol{F}}_{PK} = 2\boldsymbol{Q}_\lambda\left(\dfrac{\vec{m}}{2} + \vec{\boldsymbol{v}} \wedge n\vec{p}\right) \end{cases} \Leftrightarrow \begin{cases} \vec{\boldsymbol{F}} = q\left(\vec{E} + \vec{\boldsymbol{v}} \wedge \vec{B}\right) \end{cases}$$

Conclusion



Relativistic dynamics of the charges

Maxwell equations and Lorentz force at constant volumic expansion

Interactions of electrical type and of gravitational type between charges

String model of the dislocation line

The numerous analogies which appear *between the eulerian theory of deformable media and the theories of electromagnetism, gravitation, special relativity, general relativity and even standard model of elementary particles,* ***reinforced by the absence of particles analogue to magnetic monopoles, by a possible solution of the famous paradox of electron field energy and by the existence of a small asymmetry between curvature charges of vacancy or interstitial type****,* ***are sufficiently surprising and remarkable to alert any open and curious scientific spirit!***
But it is also clear that these analogies are, by far, not perfect.
It is then tantalizing to analyze much more carefully these analogies and to try to find how to perfect them.