II - ON THE ORIGIN OF COSMIC EXPANSION AND « DARK ENERGY » IN A LATTICE UNIVERSE

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Considering a finite imaginary sphere of a cosmic lattice, we can introduce the concept of *"cosmological evolution"* of the lattice, assuming that one injects a certain amount of kinetic energy inside the lattice. In this case, the lattice has strong temporal variations of its volume expansion, that can be modeled very simplistically assuming that volume expansion remains perfectly homogeneous throughout the lattice during its evolution.

On the cosmological behavior of a sphere of perfect solid

Let's imagine that in an absolute referential $\boldsymbol{O}\xi_1\xi_2\xi_3$, the ***GO*** observes a solid, of spherical form, of radius R_U, made of a lattice of N nodes (fig. 16.1). Let's assume this solid possesses a homogeneous volume expansion of the background with depends on time in the form

$$\tau(t) = \tau_0(t) \neq \tau_0(\vec{\xi},t) \tag{16.1}$$

In this case, the ***GO*** will observe that the radius R_U of the sphere will depend on time

$$R_U = R_U(t) \tag{16.2}$$

and that this sphere will expand or contract. This behavior will be described as a *"cosmological behaviour"* by analogy to the *theories of the cosmological expansion of the Universe.* We assume that the total energy $\boldsymbol{E}$ of the solid is a constant. It is made up of the elastic energy $\boldsymbol{F}^{él}(\tau)$ and the kinetic energy $\boldsymbol{T}(\tau)$ of expansion.

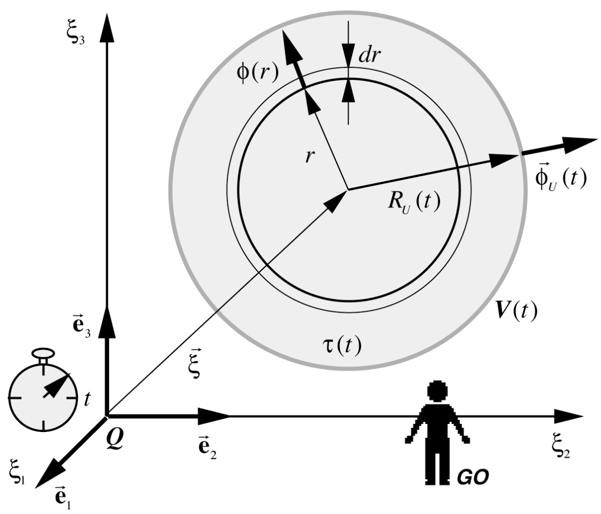


Figure 16.1 *- "Cosmological" volume expansion $\tau(t)$ of an imaginary solid sphere*

The total kinetic energy $\boldsymbol{T}$ of volume expansion is linked to the speed of expansion which we can characterize by the velocity $\vec{\phi}_U(t)$ of the surface of the sphere (fig. 16.1). The kinetic energy $\boldsymbol{T}$ can then be obtained by integrating on all the sphere and considering the energy located in the lattice contained in the volume between radius r and $r+dr$. The velocity of expansion of the surface is

$$\phi(r)=\phi_U \frac{r}{R_U} \tag{16.3}$$

since the volume expansion τ was supposed homogeneous. And using the fact that the density of sites n is given by

$$n=n_0\,\mathrm{e}^{-\tau}=\frac{N}{V_U}=\frac{3N}{4\pi R_U^3} \tag{16.4}$$

we have for the kinetic energy

$$\boldsymbol{T}=\int_0^{R_U}\frac{1}{2}mn\phi^2(r)4\pi r^2dr=\frac{3Nm\phi_U^2}{2R_U^5}\int_0^{R_U}r^4\,dr=\frac{3}{10}Nm\phi_U^2 \tag{16.5}$$

We deduce that the velocity of expansion ϕ_U is proportional to $\sqrt{\boldsymbol{T}}=\sqrt{\boldsymbol{E}-\boldsymbol{F}^{él}}$

$$\phi_U(\tau)=\sqrt{\frac{10}{3Nm}\boldsymbol{T}(\tau)}=\sqrt{\frac{10}{3Nm}\left(\boldsymbol{E}-\boldsymbol{F}^{él}(\tau)\right)} \tag{16.6}$$

We also deduce that

$$\frac{d\phi_U(\tau)}{d\tau}=-\frac{5}{3Nm}\frac{d\boldsymbol{F}^{él}(\tau)/d\tau}{\phi_U(\tau)} \tag{16.7}$$

and so, as a consequence, the derivative $d\phi_U/d\tau$ tends towards $\pm\infty$ if ϕ_U goes to zero and $d\boldsymbol{F}^{él}/d\tau$ is finite and not null. Let's study the cosmological behavior of perfect solids and cosmic lattices that we have previously defined.

The cosmological evolution of a cosmic lattice with $K_0=0$ and $K_1>0$

Imagine that in the absolute framework $\boldsymbol{O}\xi_1\xi_2\xi_3$, the ***GO*** observes a *cosmic lattice* of spherical radius R_U, consisting of a lattice with N meshes (fig. 16.1). The elastic energy $\boldsymbol{F}^{él}$ is written, by using *(13.7)*

$$\boldsymbol{F}^{él}=Nf^{él}=\underbrace{-\frac{NK_0}{n}\tau}_{K_0=0}+\frac{NK_1}{n}\tau^2+\underbrace{\frac{NK_2}{n}\sum_i(\vec{\alpha}_i^{él})^2}_{\vec{\alpha}_i^{él}=0}+\underbrace{\frac{NK_3}{n}(\vec{\omega}^{él})^2}_{\vec{\omega}^{él}=0}=\frac{NK_1}{n}\tau^2=\frac{NK_1}{n_0}\tau^2\,\mathrm{e}^{\tau} \tag{16.9}$$

To plot the behavior of $\boldsymbol{F}^{él}$ as a function of τ for this lattice, we must look for the extremas of $\boldsymbol{F}^{él}(\tau)$

$$\frac{d\boldsymbol{F}^{él}}{d\tau}=\frac{NK_1}{n_0}(2+\tau)\tau\,\mathrm{e}^{\tau}=0 \quad\Rightarrow\quad \tau=0,-2\ et\ -\infty \tag{16.10}$$

If the modulus K_1 of the lattice is positive, then for $\tau=0$ and $\tau\to-\infty$, the value of $\boldsymbol{F}^{él}$ tends towards two minimas equal to zero, while for $\tau=-2$, the value of $\boldsymbol{F}^{él}$ goes through a minimum equal to $\boldsymbol{F}^{él}_{max}=4\,\mathrm{e}^{-2}NK_1/n_0\cong 0{,}54NK_1/n_0$. The graph of $\boldsymbol{F}^{él}(\tau)$ is shown in figure 16.4a, and we can see that it is very different from the case of a lattice of usual perfect solid.

There are three modes of oscillation depending on the value of $\boldsymbol{E}$, as illustrated in figure 16.4:

- if $\boldsymbol{E} \leq \boldsymbol{F}^{él}_{max} = 4\,\mathrm{e}^{-2} NK_1 / n_0$, there are two modes of oscillation possible, a first mode between $\tau \to -\infty$ and $\tau_1 < 0$ and a second mode between $\tau_2 < 0$ and $\tau_3 > 0$,
- if $\boldsymbol{E} \geq \boldsymbol{F}^{él}_{max} = 4\,\mathrm{e}^{-2} NK_1 / n_0$, there is a third mode of oscillation possible, between $\tau \to -\infty$ and $\tau_4 > 0$.

In the graphs of figure 16.4, we can show the limit $\tau_{0cr} = -1 - 2K_2 / 3K_1 < -1$ taken from relation *(14.26)*. We show a value of τ_{0cr} close to -1, corresponding to the case where $K_1 >> K_2$. There are again domains of different behaviors of the solid: a domain where there coexists transverse and longitudinal waves (for $\tau \geq \tau_{0cr}$) and a domain where there are only transverse waves and localized vibrational eigenmodes (for $\tau \leq \tau_{0cr}$). But unlike the perfect solid, in the *cosmic lattice*, the position of these domains is reversed along the τ axis!

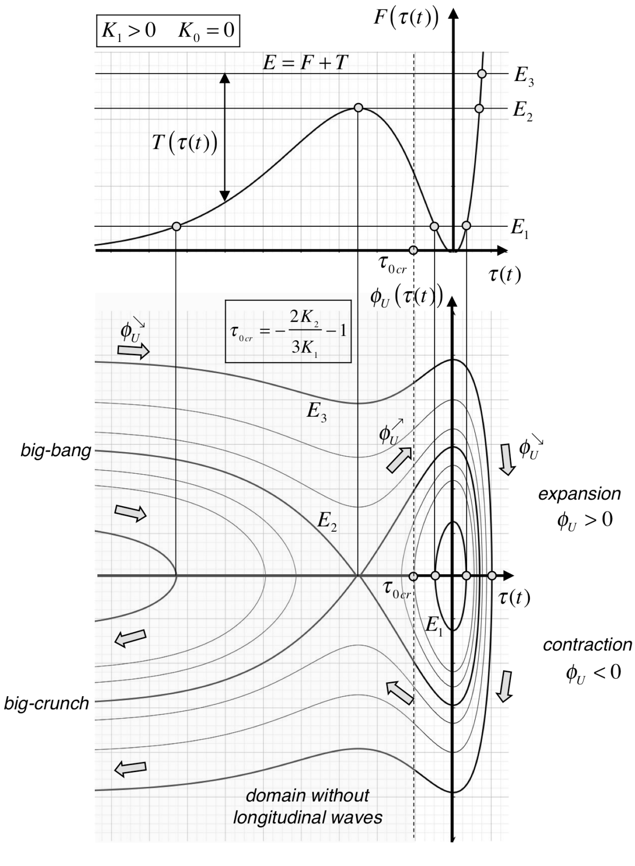


Figure 16.4 *- "cosmological" behavior of elastic energy $\boldsymbol{F}^{él}(\tau)$ of expansion and velocity $\vec{\phi}_U(\tau)$ of expansion of an imaginary cosmic lattice with $K_1 > 0$*

The cosmological evolution of a cosmic lattice with $K_0 = 0$ and $K_1 < 0$

If the modulus K_1 of the lattice is negative, the plastic energy $\boldsymbol{F}^{él}(\tau)$ presents two maximas for $\tau \to -\infty$ and $\tau = 0$, and a minima for $\tau = -2$. Furthermore, $\boldsymbol{F}^{él}(\tau) \to -\infty$ for $\tau \to \infty$

(fig. 16.5). We can consider here three different cases following the value of total energy $\boldsymbol{E}$:

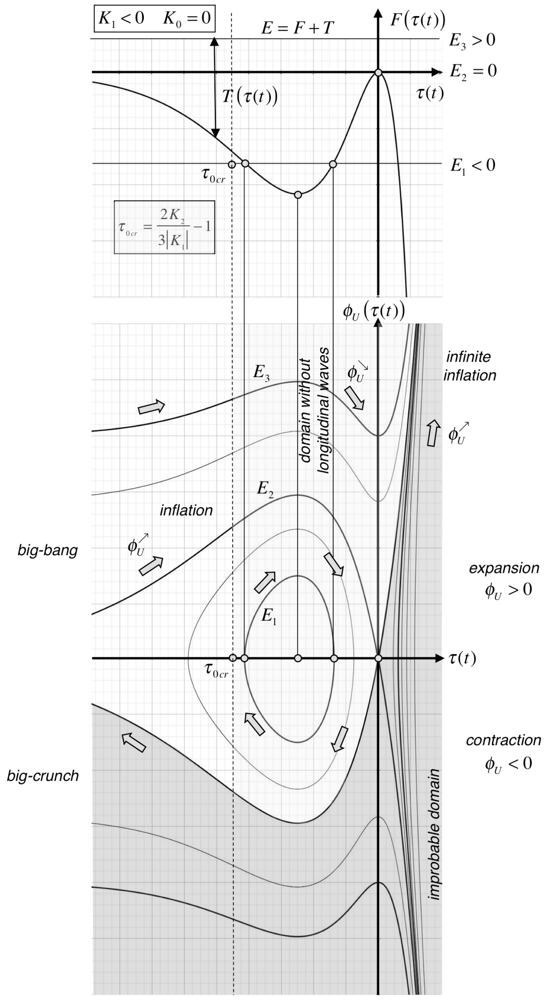


Figure 16.5 *- "cosmological behavior" of elastic energy* $\boldsymbol{F}^{él}(\tau)$ *of expansion and velocity* $\vec{\phi}_U(\tau)$ *of expansion of an imaginary cosmic lattice with* $K_1 < 0$

- if $\boldsymbol{E} < 0$, there exists a "*cosmological solution*" around the value $\tau = -2$, for which the lattice contacts and expands indefinitely between values $\tau_1 < 0$ and $\tau_2 < 0$, and a second solution for which the lattice dilates indefinitely, at constant velocity, from the value $\tau_3 > 0$,

- if $\boldsymbol{E} = 0$, there exists a solution for values inferior to $\tau = 0$, and a solution for the superior values. The lattice can dilate from $\tau \to -\infty$, and can afterwards, either contract towards $\tau \to -\infty$ and start the cycle again, or dilate indefinitely towards $\tau \to +\infty$,

- if $\boldsymbol{E} > 0$, there exists a unique solution for which the lattice dilates one time only from $\tau \to -\infty$ to $\tau \to +\infty$. The symmetric solution which would consist in the lattice contracting form $\tau = +\infty$ *,* where the lattice possesses a phenomenal kinetic energy of contraction, to $\tau \to -\infty$ is not prohibited but it is seems strongly improbable!

In the case of this lattice, we also notice the existence of domains of different behavior: for $\tau > \tau_{0cr} = -1 + 2K_2 / 3|K_1| > -1$ a domain where there are both transverse and longitudinal waves, and for $\tau < \tau_{0cr} = -1 + 2K_2 / 3|K_1| > -1$ a domain where there are only transverse waves and localized longitudinal eigenmodes of vibration.

The cosmological evolution of a cosmic lattice with $K_1 = 0$ and $K_0 < 0$

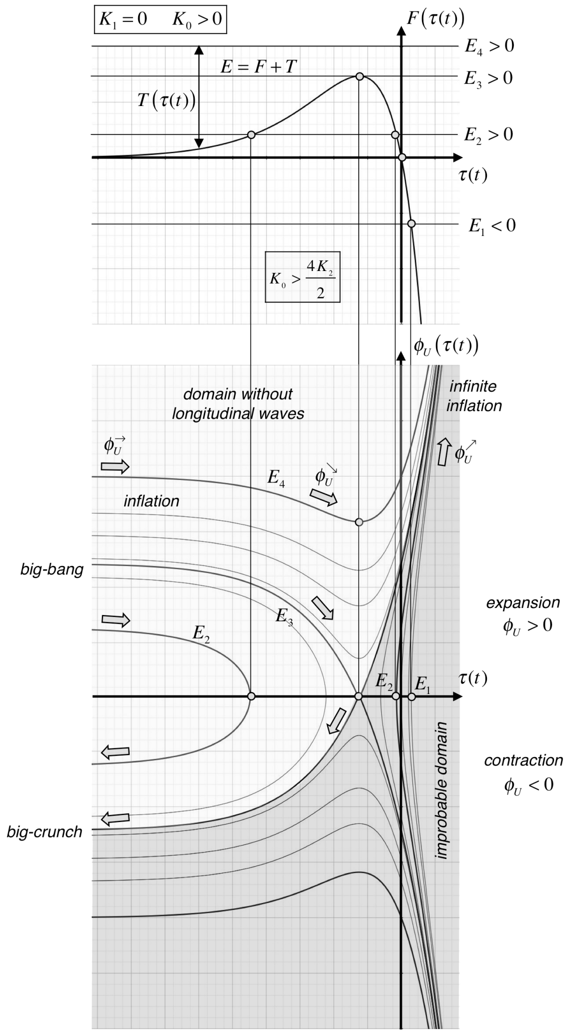


Figure 16.6 *- "cosmological behavior" of elastic energy $\boldsymbol{F}^{él}(\tau)$ of expansion and velocity $\vec{\phi}_U(\tau)$ of expansion of an imaginary cosmic lattice with $K_0 < 0$*

If, in the absolute referential $\boldsymbol{O}\xi_1\xi_2\xi_3$ of the ***GO*** we consider a cosmic lattice with $K_1 = 0$ and $K_0 < 0$**,** the elastic energy of expansion $\boldsymbol{F}^{él}(\tau)$ is written

$$\boldsymbol{F}^{él} = Nf^{él} = -\frac{NK_0}{n}\tau + \underbrace{\frac{NK_1}{n}\tau^2}_{K_1=0} + \underbrace{\frac{NK_2}{n}\sum_i(\vec{\alpha}_i^{él})^2}_{\vec{\alpha}_i^{él}=0} + \underbrace{\frac{NK_3}{n}(\vec{\omega}^{él})^2}_{\vec{\omega}^{él}=0} = -\frac{NK_0}{n}\tau = -\frac{NK_0}{n_0}\mathrm{e}^{\tau}\,\tau \qquad (16.11)$$

To plot the behavior of $\boldsymbol{F}^{él}(\tau)$ as a function of τ for this solid, we must seek the extremes of $\boldsymbol{F}^{él}(\tau)$

$$\frac{d\boldsymbol{F}^{él}(\tau)}{d\tau} = -\frac{NK_0}{n_0}(1+\tau)\mathrm{e}^{\tau} = 0 \quad \Rightarrow \quad \tau = -1 \quad et \quad \tau \to -\infty \qquad (16.12)$$

If the modulus K_0 *of the solid is negative,* the energy $\boldsymbol{F}^{él}(\tau)$ as a function of τ presents a minimum for $\tau = -1$ as illustrated in figure 16.6.
We then deduce the "cosmological behavior" of this type of lattice and show it in figure 16.6:
- if $\boldsymbol{E} < 0$, the lattice oscillates indefinitely on a closed trajectory between minimum $\tau_{\min}$ and maximum $\tau_{\max}$,
- if $\boldsymbol{E} \geq 0$, the lattice oscillates between $\tau \to -\infty$ and a maximum $\tau_{\max}$.

In this case the lattice still presents longitudinal waves as since $K_0 < 0$, we also have $K_0 < 4K_2 / 3$.

The cosmological evolution of a cosmic lattice with $K_1 = 0$ ***and*** $K_0 > 0$

If the K_0 *modulus of a cosmic lattice is positive,* the energy $E^{él}(\tau)$ as a function of τ presents a maximum for $\tau = -1$ as illustrated in figure 16.7.

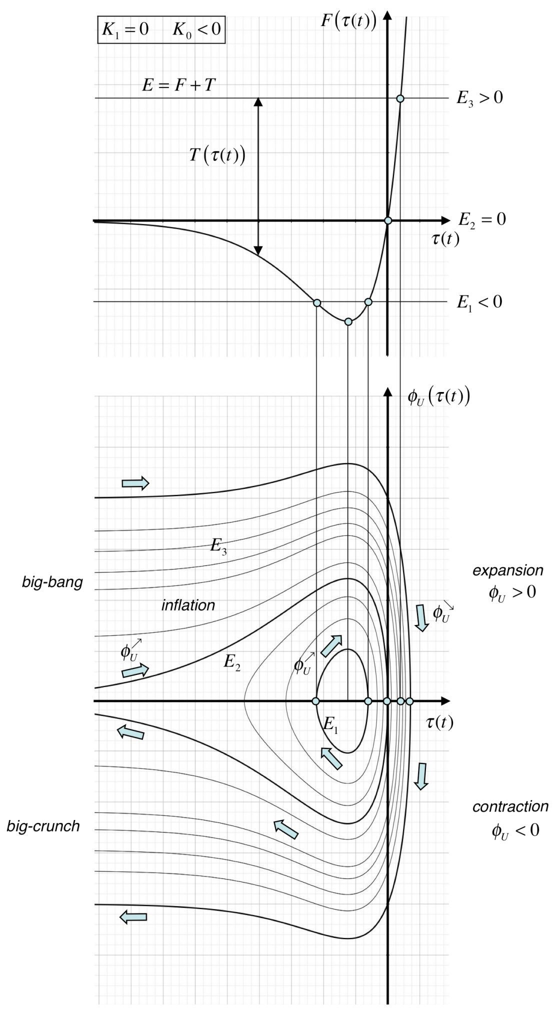


Figure 16.7 *- "cosmological behavior" of elastic energy* $\boldsymbol{F}^{él}(\tau)$ *of expansion and velocity* $\vec{\phi}_U(\tau)$ *of expansion of an imaginary cosmic lattice with* $K_0 > 0$

We deduce the "cosmological behavior" of this type of lattice as shown in figure 16.7:

- if $0 < \boldsymbol{E} < \boldsymbol{F}^{él}_{max} = NK_0 / \mathrm{e} n_0$, the lattice presents two possible trajectories, one that oscillates indefinitely between $\tau \to -\infty$ and a maximal value $\tau_{\max}$, and one which corresponds to an irreversible expansion, at constant velocity, from an initial value with $\vec{\phi}_U = 0$,

- if $\boldsymbol{E} \geq \boldsymbol{F}^{él}_{max} = NK_0 / \mathrm{e} n_0$, the lattice presents an irreversible expansion from $\tau = -\infty$ to $\tau \to +\infty$, with a first decreasing velocity and then increasing velocity,

- if $\boldsymbol{E} \leq 0$, the evolution of the lattice only has one trajectory presenting an irreversible expansion from value $\tau_{\min} > 0$ to $\tau \to +\infty$, with an increasing velocity.

In this case, the lattice presents longitudinal waves if $0 < K_0 < 4K_2 / 3$, but does not present them if $K_0 > 4K_2 / 3$.

The cosmological evolution of a cosmic lattice with $K_0 > 0$ ***and*** $K_1 > 0$

The elastic free energy of this lattice is written

$$\boldsymbol{F}^{él}(\tau) = Nf^{él} = N\left(-\frac{K_0}{n}\tau + \frac{K_1}{n}\tau^2\right) = \frac{N}{n_0}\left(K_1\tau - K_0\right)\tau\,\mathrm{e}^{\tau} \tag{16.13}$$

This function is represented at the top of figure 16.8. It has zeroes for

$$\boldsymbol{F}^{él}(\tau) = 0 \quad \Leftrightarrow \quad \tau = \begin{cases} 0 \\ K_0 / K_1 \\ \to -\infty \end{cases} \tag{16.14}$$

as well as a maximum in the domain $\tau < 0$ and a minimum in the domain $\tau > 0$. According to relation *(16.6),* these extremas correspond respectively to the minimum and maximum of the velocity of expansion $\phi_U(\tau)$ of the lattice, so that we have

$$\frac{d\boldsymbol{F}^{él}(\tau)}{d\tau} = 0 \Rightarrow \begin{cases} \tau_{\boldsymbol{F}^{él}\max} = \tau_{\phi_U \min} = \left(\dfrac{K_0}{2K_1} - 1\right) - \sqrt{\dfrac{K_0}{2K_1}\left(\dfrac{K_0}{2K_1} + 1\right)} \underset{\frac{K_0}{K_1} >> 1}{\longrightarrow} -\dfrac{3}{2} \\ \tau_{\boldsymbol{F}^{él}\min} = \tau_{\phi_U \max} = \left(\dfrac{K_0}{2K_1} - 1\right) + \sqrt{\dfrac{K_0}{2K_1}\left(\dfrac{K_0}{2K_1} + 1\right)} \underset{\frac{K_0}{K_1} >> 1}{\longrightarrow} \dfrac{K_0}{K_1} - \dfrac{1}{2} \end{cases} \tag{16.15}$$

We deduce the “cosmological behavior” of this type of lattice, as shown in figure 16.8:

- if $\boldsymbol{E} < 0$, the lattice presents only one possible trajectory, entirely in the domain $\tau > 0$, and which corresponds to a contraction and an expansion that keeps on going between two extreme values of τ,

- if $0 < \boldsymbol{E} < \boldsymbol{F}^{él}_{max}$, the lattice presents two possible trajectories: the first one is an expansion/contraction that goes on indefinitely between a positive and a negative value of τ, and the second corresponds to an indefinite oscillation between a negative value of τ and an expansion going to $\tau \to -\infty$,

- if $\boldsymbol{E} > \boldsymbol{F}^{él}_{max}$, the lattice presents only one trajectory which is rather interesting. We oscillate indefinitely between a *big-bang* and a *big crunch*! The big-bang is followed by an expansion phase which is very fast, then a slowdown, and then again an expansion with increasing velocity, and suddenly an inversion of the velocity of expansion, so it contracts by retracing all the steps followed during the expansion phase. The contraction finishes with a big crunch, which can only be followed by a big-bang since the lattice has accumulated a total kinetic energy T equal to E, this phenomena is called "*big bounce"!*

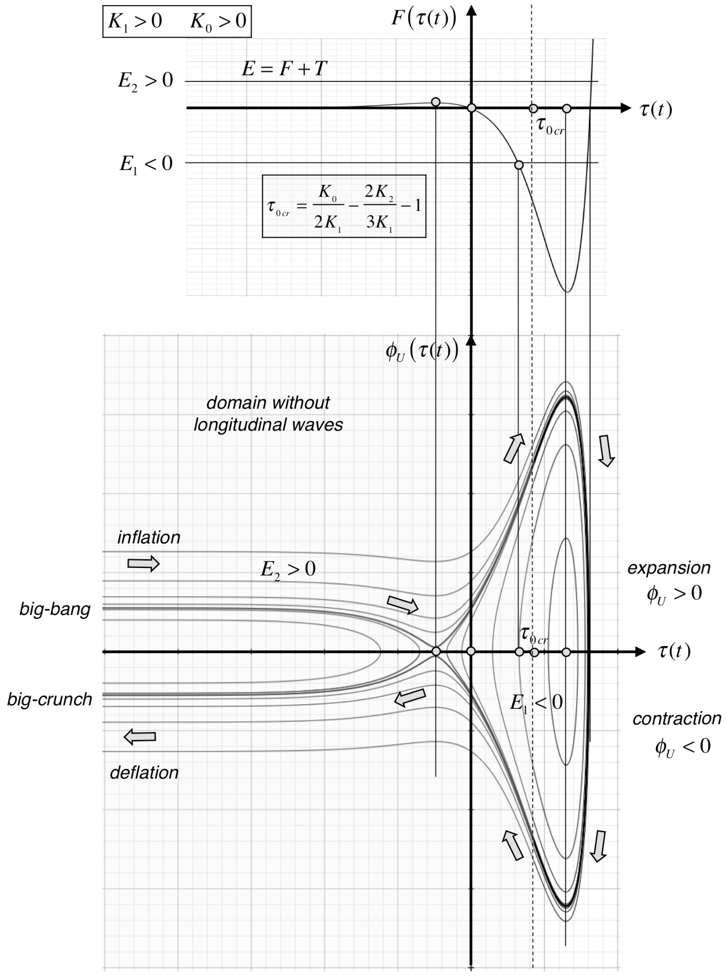


Figure 16.8 *- «cosmological behavior» of elastic energy $F^{él}(\tau)$ of expansion and velocity $\vec{\phi}_U(\tau)$ of expansion of an imaginary cosmic lattice with $K_0 > 0$ and $K_1 > 0$*

In the case of this lattice, we notice too the existence of domains of volume expansion that present different behaviors with regards to longitudinal waves: a domain where we have transverse and longitudinal waves for $\tau > \tau_{0cr} = K_0 / 2K_1 - 2K_2 / 3K_1 - 1$, and a domain for $\tau < \tau_{0cr} = K_0 / 2K_1 - 2K_2 / 3K_1 - 1$ where there are only transverse waves and localized vibrational eigenmodes. The domain where there are no longitudinal waves corresponds precisely to the domain of the big-bang, the inflation, the slowdown of inflation, finally followed by an acceleration of the expansion!

On the analogy with the cosmological evolution of our universe

In figure 16.9, we show eight different behaviors that can be obtained with a cosmic lattice, depending on the values that the moduli K_0 and K_1 can take. It is also shown in this figure the domains of expansion in which the longitudinal waves cannot exist.

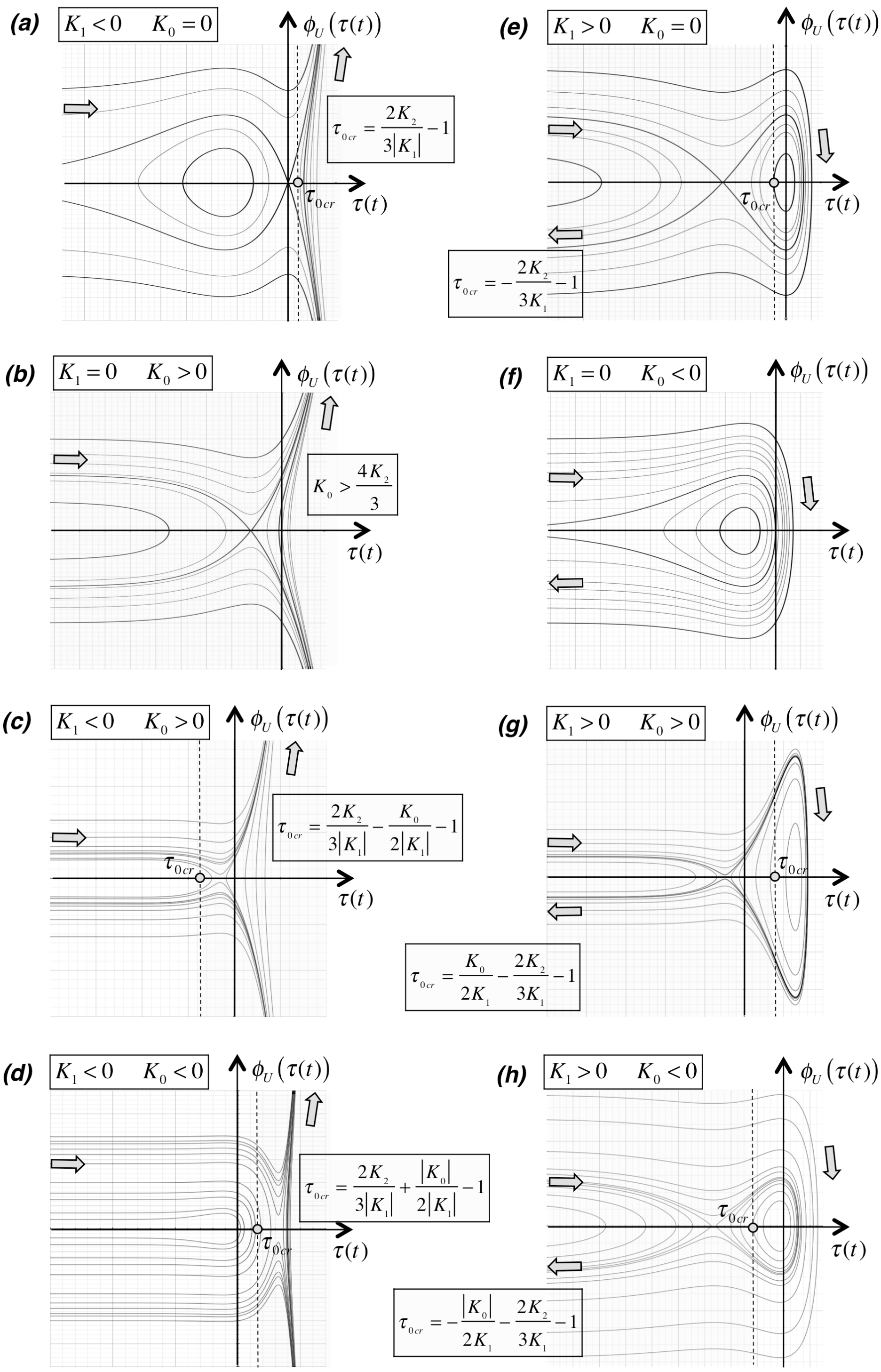


***Figure 16.9** - all the "cosmological behaviors" that are possible for cosmic lattices, depending on values K_0 and K_1: **(a)** through **(d)** the lattices with infinite accelerating expansion, **(e)** through **(h)** the lattices oscillating from big-bang to big-crunch*

It is noted that there are four different *"cosmological behavior*", three of which have convincing analogies with the cosmology of the real universe:

- cosmological lattices with $K_1 < 0$ which are reported in figures 16.9 (a), (c) and (d). These three types of lattices all have a big bang followed by high speed inflation, a slowdown in inflation and ultimately an expansion at increasing velocity towards $\tau \to +\infty$. All the stages follow in perfect order. The disappearance of the longitudinal waves takes place in these networks to higher expansions than a critical value τ_{0cr}, which depends on the value of the shear modulus $K_2 > 0$,

- the cosmic lattice of figure 16.9 (b), with $K_1 = 0$ and $K_0 > 0$ for which there are never longitudinal waves provided that $K_0 > 4K_2 / 3$, making it a very simple and very interesting case to describe the cosmological behavior of the real universe,

- the cosmic lattice with $K_1 > 0$ or $K_1 = 0$ and $K_0 < 0$ are shown in figures 16.9 (e), (g) and (h). These three types of lattice go through the four stages of the cosmology of the real universe, in the absence of longitudinal waves (a "big bang" from a singularity of space-time, followed by a period of very rapid inflation and a slowdown in inflation, followed by an expansion whose speed seems to increase over time), before entering an expansion phase during which the longitudinal waves appear, and precede a symmetrical contraction phase back to the singularity state $\tau \to -\infty$ ("big crunch"). In this case, there is a region of the diagram for which $\tau < \tau_{0cr}$ where there are no longitudinal waves, and wherein the lattice is expanding with increasing velocity. Note that the lattice of figure 16.9 (g) could be an excellent candidate to describe the cosmological behavior of the real universe, because all its elastic moduli are positive,

- finally, the cosmic lattice of figure 16.9 (f), with $K_1 = 0$ and $K_0 < 0$, does not present the stages corresponding to the cosmology of the real universe, and it always has longitudinal waves. It is clearly not suitable to describe the cosmological behavior of our universe.

The "cosmological behavior" of a cosmic lattice can be illustrated more clearly by plotting the velocity of volume expansion $d\tau / dt$ as a function of the volume expansion τ, as shown in the cases (c) and (d) with $K_1 < 0$ in figure 16.10 and for the case (g) and (h) with $K_1 > 0$ in figure 16.11. To find these behaviors, we retrieve the value R_U of the expression *(16.4)*

$$R_U = \left(\frac{3N}{4\pi n_0} \right)^{1/3} \mathrm{e}^{\tau/3} \tag{16.16}$$

and we deduce the velocity of expansion $\phi_U(\tau)$

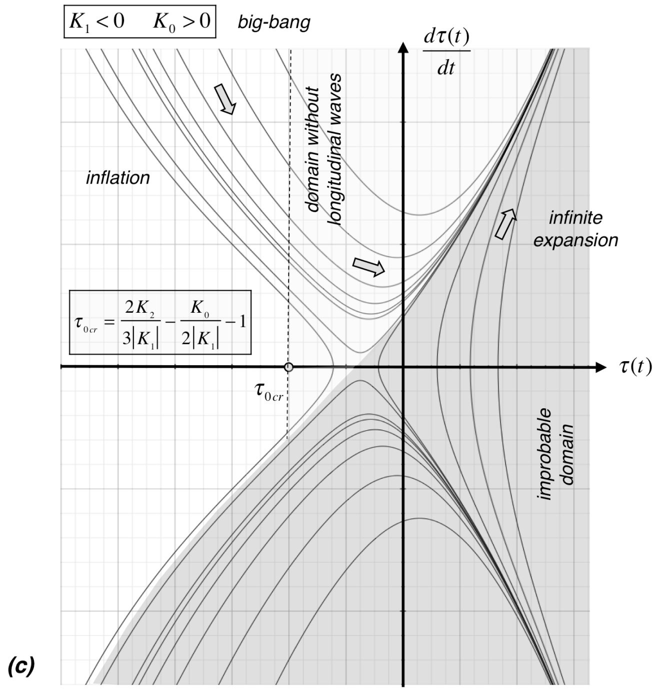
$$\phi_U(\tau) = \frac{dR_U}{dt} = \frac{1}{3}\left(\frac{3N}{4\pi n_0} \right)^{1/3} \mathrm{e}^{\tau/3} \frac{d\tau}{dt} \tag{16.17}$$

which, compared to expression *(16.6) of* $\phi_U(\tau)$ allows us to write

$$\frac{d\tau}{dt} = 3\left(\frac{4\pi n_0}{3N} \right)^{1/3} \mathrm{e}^{-\tau/3} \phi_U(\tau) = 3\left(\frac{4\pi n_0}{3N} \right)^{1/3} \mathrm{e}^{-\tau/3} \sqrt{\frac{10}{3Nm}\left(\boldsymbol{E} - \boldsymbol{F}^{él}(\tau) \right)} \tag{16.18}$$

The behavior of the rate of volume expansion $d\tau / dt$ as a function of τ can then be deduced from the knowledge of $\boldsymbol{F}^{él}(\tau)$, which allows us to do the plots of figures 16.10 and 16.11.

The figures 16.10 and 16.11 are very interesting because they clearly show the existence of an extremely fast initial stage of inflation of the volume expansion in cosmic lattices since $d\tau / dt \to \pm\infty$ for $\tau \to -\infty$ just after the big bang stage or just before the big crunch, and the rate of expansion or contraction of the volume is at a minimum before accelerating again, just after the stage of inflation or just after the stage of re-contraction.



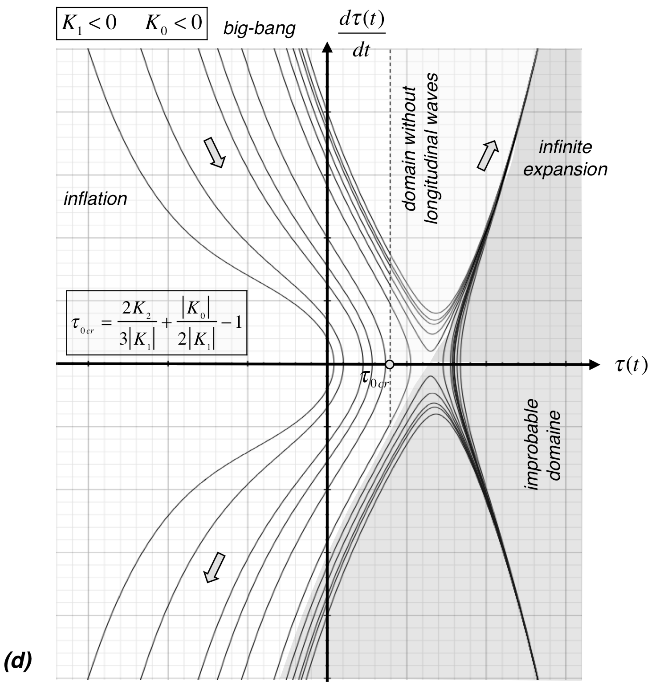


Figure 16.10 *- «cosmological behaviors» of the velocity $d\tau / dt$ of expansion as a function of expansion τ of two imaginary cosmic lattices with $K_1 < 0$*

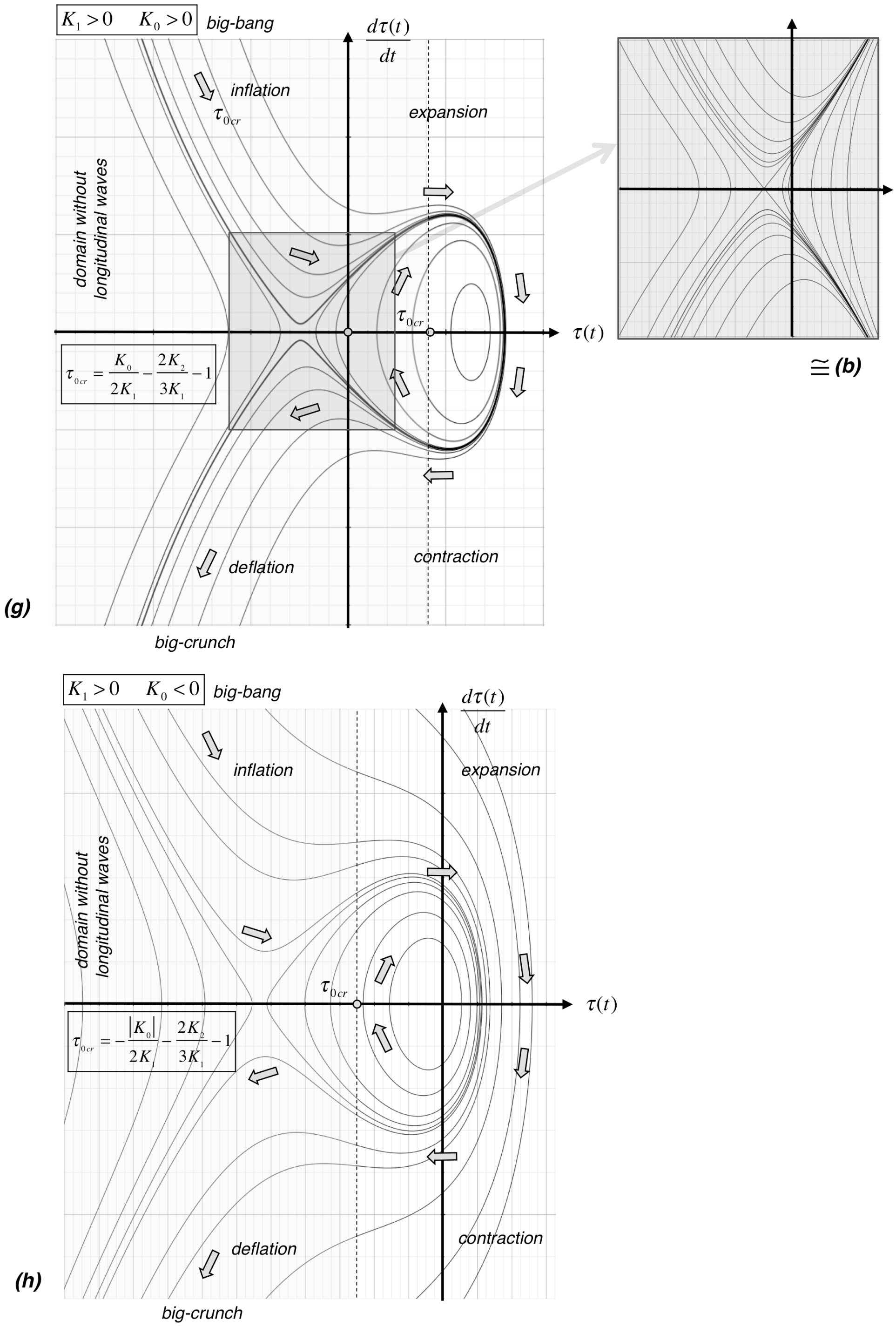


***Figure 16.11** - «cosmological behaviors» of the velocity $d\tau / dt$ of expansion as a function of expansion τ of two imaginary cosmic lattices with $K_1 > 0$*

The figures 16.10 and 16.11 are very interesting because they clearly show the existence of an extremely fast initial stage of inflation of the volume expansion in cosmic lattices since $d\tau / dt \to \pm\infty$ for $\tau \to -\infty$ just after the big bang stage or just before the big crunch, and the rate of expansion or contraction of the volume is at a minimum before accelerating again, just after the stage of inflation or just after the stage of re-contraction.

On the limits of our model

It goes without saying that the modeling used in this chapter to describe the "cosmological behaviors" of imaginary lattices is extremely simple, if not simplistic. It is essentially the initial assumption of a homogeneous volume expansion throughout the lattice that can be questioned, because with this hypothesis was evaded the two major problems that would lead in principle to much more complicated models: the fact that the solid is subjected to Newtonian dynamics in the absolute space of ***GO***, and the fact that we should have put a condition on the validity of the pressure at the outer edge of the solid sphere. But despite the extreme simplifications of our modeling, the overall predicted behaviors in figures 16.9 to 16.11 should still remain close enough to the behaviors which could have been obtained by a more realistic treatment of the problem!

On the 'reasonable' choice of a cosmic lattice to describe the real universe

Among the various lattices proposed in this chapter, it is clear that the cosmic lattices have more interesting features than the perfect solids to describe the experimental observations of cosmologists. It is obviously not possible here to choose the cosmic lattice which is close to most of the known cosmological evolution of the real universe. But from a philosophical point of view and from the point of view of common sense, the cosmic lattices (fig. 16.9 (e) to (h)) which have a big bang followed by a big crunch, and thus ultimately a big-bounce, are much more satisfying for a Cartesian mind than cosmic lattices presenting a single and infinite expansion (fig. 16.9 (a) to (d)). One can then emit here a conjecture of 'philosophical nature'

Conjecture 3: *It seems more 'reasonable' to imagine cosmic lattices with $K_1 > 0$,
so as to have a finite expansion* *(16.19)*

As for value of K_0, nothing allows us for the moment to propose a positive, zero or negative value, as the cases illustrated in figure 16.11 are both very interesting!

On the origin of the 'dark energy"

It should finally be noted that the elastic energy $\boldsymbol{F}^{él}(\tau)$ contained in the cosmic lattice could very well correspond to the 'dark energy' which astrophysicists introduce to explain the acceleration of the velocity of expansion of the universe which was recently observed experimentally, since it is that elastic energy which is fully responsible for an increase of the rate of volume expansion via relation *(16.18)*.