III - ON THE ORIGIN OF MAXWELL'S EQUATIONS OF ELECTROMAGNETISM IN A LATTICE UNIVERSE

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We start by showing that we can separate the field of volume expansion from the other fields in the Newton equation of a cosmic lattice in the case where the concentration of point defects are constant. Then we use these results to obtain the Maxwell's equations of evolution of a lattice in the case where the volume expansion can be treated as constant.

On the separability of Newton's equation of a cosmic lattice in a 'rotational' part and a 'divergent' part

Assume that the field of volume expansion in the cosmic lattice is represented by a homogenous background field τ_0 on which we superpose a field of elastic expansion $\tau^{él}$

Hypothesis 1: $\tau = \left(\tau_0 + \tau^{él}\right)$ *(17.1)*

Introduce this field in the equation of Newton *(13.9)*. We have

$$n\frac{d\vec{p}}{dt} = -2\left(K_2+K_3\right)\overrightarrow{\mathrm{rot}}\,\vec{\omega}^{él} + \overrightarrow{\mathrm{grad}}\left[\left(\frac{4}{3}K_2+2K_1\left(1+\tau_0\right)-K_0\right)\tau^{él}+K_1\left(\tau^{él}\right)^2+F^{rot}\right]+2K_2\vec{\lambda}+nm\vec{\phi}_I\frac{dC_I}{dt}-nm\vec{\phi}_L\frac{dC_L}{dt} \tag{17.2}$$

in which F^{rot} represents the density of energy of deformation by elastic and anelastic shear strains and rotations, and is worth

$$F^{rot} = K_2\sum_i(\vec{\alpha}_i^{él})^2 + K_1^{an}\sum_i(\vec{\alpha}_i^{an})^2 + 2K_3(\vec{\omega}^{él})^2 + 2K_2^{an}(\vec{\omega}^{an})^2 \tag{17.3}$$

Suppose further than the atomic concentrations of vacancies and interstitials are homogeneous constants in the lattice such that

Hypothesis 2: $\dfrac{dC_I}{dt} = \dfrac{dC_L}{dt} = 0$ *(17.4)*

In this case the equation of Newton simplifies into

$$n\frac{d\vec{p}}{dt} = -2\left(K_2+K_3\right)\overrightarrow{\mathrm{rot}}\,\vec{\omega}^{él} + \overrightarrow{\mathrm{grad}}\left[\left(\frac{4}{3}K_2+2K_1\left(1+\tau_0\right)-K_0\right)\tau^{él}+K_1\left(\tau^{él}\right)^2+F^{rot}\right]+2K_2\vec{\lambda} \tag{17.5}$$

in which the quantity of movement can be written according to *(5.101) and (5.78)*

$$\begin{cases} \vec{p} = m\vec{\phi} + m\left(C_I - C_L\right)\vec{\phi} + m\left(\vec{J}_I - \vec{J}_L\right)/n \\ \vec{J}_L = nC_L\left(\vec{\phi}_L - \vec{\phi}\right) \\ \vec{J}_I = nC_I\left(\vec{\phi}_I - \vec{\phi}\right) \end{cases} \tag{17.6}$$

Thanks to the second hypothesis, the linearity of equations *(17.6)* with respect to the various velocities, means that it is possible to separate the equations in two different sets by separating the velocities $\vec{\phi}$, $\vec{\phi}_L$ and $\vec{\phi}_I$ in a component indexes *«rot»*, associated with the deformations by shear and rotation on one hand, and a component indexed by *«div»*, associated with the deformations by volume expansion on the other hand. We write

$$\vec{\phi} = \vec{\phi}^{rot} + \vec{\phi}^{div} \quad ; \quad \vec{\phi}_L = \vec{\phi}_L^{rot} + \vec{\phi}_L^{div} \quad ; \quad \vec{\phi}_I = \vec{\phi}_I^{rot} + \vec{\phi}_I^{div} \tag{17.7}$$

We also have two contributions to the equation of Newton:

- *a contribution which pilots the elastic fields of shear and rotation,* via the vectorial field of rotation $\vec{\omega}^{él}$. This contribution only depends on volume expansion τ via the presence of the density of sites $n = n_0\, \mathrm{e}^{-(\tau_0+\tau^{él})}$, and it is written

$$n\frac{d\vec{p}^{rot}}{dt} = -2\left(K_2 + K_3\right)\overrightarrow{\mathrm{rot}}\,\vec{\omega}^{él} + 2K_2\vec{\lambda}^{rot} \tag{17.8}$$

$$\text{with} \quad \begin{cases} \vec{p}^{rot} = m\vec{\phi}^{rot} + m\left(C_I - C_L\right)\vec{\phi}^{rot} + m\left(\vec{J}_I^{rot} - \vec{J}_L^{rot}\right)/\,n \\ \vec{J}_L^{rot} = nC_L\left(\vec{\phi}_L^{rot} - \vec{\phi}^{rot}\right) = nC_L\Delta\vec{\varphi}_L^{rot} \\ \vec{J}_I^{rot} = nC_I\left(\vec{\phi}_I^{rot} - \vec{\phi}^{rot}\right) = nC_I\Delta\vec{\varphi}_I^{rot} \end{cases} \tag{17.9}$$

- *a contribution which pilots the field of perturbation of volume expansion*, and which depends on the previous solution via the density of energy F^{rot} of deformation by elastic and anelastic shear strains and rotations, and which is written

$$n\frac{d\vec{p}^{div}}{dt} = \overrightarrow{\mathrm{grad}}\left[\left(\frac{4}{3}K_2 + 2K_1\left(1+\tau_0\right) - K_0\right)\tau^{él} + K_1\left(\tau^{él}\right)^2 + F^{rot}\right] + 2K_2\vec{\lambda}^{div} \tag{17.10}$$

$$\text{with} \quad \begin{cases} F^{rot} = K_2\sum_i(\vec{\alpha}_i^{él})^2 + K_1^{an}\sum_i(\vec{\alpha}_i^{an})^2 + 2K_3(\vec{\omega}^{él})^2 + 2K_2^{an}(\vec{\omega}^{an})^2 \\ \vec{p}^{div} = m\vec{\phi}^{div} + m\left(C_I - C_L\right)\vec{\phi}^{div} + m\left(\vec{J}_I^{div} - \vec{J}_L^{div}\right)/\,n \\ \vec{J}_L^{div} = nC_L\left(\vec{\phi}_L^{div} - \vec{\phi}^{div}\right) = nC_L\Delta\vec{\varphi}_L^{div} \\ \vec{J}_I^{div} = nC_I\left(\vec{\phi}_I^{div} - \vec{\phi}^{div}\right) = nC_I\Delta\vec{\varphi}_I^{div} \end{cases} \tag{17.11}$$

The density of flexion charges was also separated in two parts: the *charges of rotational flexion* and the *charges of divergent flexion,* which satisfy the following relations

$$\vec{\lambda} = \vec{\lambda}^{rot} + \vec{\lambda}^{div} \quad \text{such that} \quad \overrightarrow{\mathrm{rot}}\,\vec{\lambda}^{rot} \neq 0 \quad \text{and} \quad \mathrm{div}\,\vec{\lambda}^{div} = \theta \tag{17.12}$$

They connect the Newton's equation for expansion $\tau^{él}$ *(17.10)* to the density of charge of curvature θ within the lattice.

This split of Newton's equation, in the case where concentrations of interstitials and vacancies are homogeneous constants allows us, with equations of table 11.2, to resolve the spatiotemporal evolution problems of the generalized perfect lattice, separating the solving of fields of elastic shear and rotation from the solving of the volume expansion of the lattice. With additional simplifying assumptions, it is possible to solve completely these two sets of equations. This is what we will show in the next section, considering the particular case where the volume expansion field can be considered almost constant.

On the maxwellian behavior of the rotational part

Now make the assumption that the average value of the volume expansion $\langle\tau\rangle=\tau_0+\langle\tau^{él}\rangle$ in the cosmological lattice can be considered in first approximation as a homogeneous constant, so that the site density n may also be regarded on average as a constant

Hypothesis 3: $\langle\tau\rangle=\tau_0+\langle\tau^{él}\rangle\cong cste \quad\Rightarrow\quad n\cong\langle n\rangle\cong n_0\,\mathrm{e}^{\tau_0+\langle\tau^{él}\rangle}\cong cste$ *(17.13)*

With these hypothesis, we can re-write the equations of Newton *(17.8)* by introducing a *vectorial moment* $\vec{m}$ conjugated to rotations $\vec{\omega}^{él}$, under the form

$$\frac{d(n\vec{p}^{\,rot})}{dt}=-2\left(K_2+K_3\right)\overrightarrow{\mathrm{rot}}\,\vec{\omega}^{él}+2K_2\vec{\lambda}^{rot}=-\frac{1}{2}\overrightarrow{\mathrm{rot}}\,\vec{m}+2K_2\vec{\lambda}^{rot} \qquad (17.15)$$

By hypothesis, the *anelasticity of the lattice* manifests itself purely by shear and/or rotation, so that it can be represented here by a vector of anelastic rotation $\vec{\omega}^{an}$, by writing the relation *(2.40)* under the form

$$\vec{\omega}^{\delta}=\vec{\omega}+\vec{\omega}_0(t)=\vec{\omega}^{él}+\vec{\omega}^{an}+\vec{\omega}_0(t)=\frac{1}{4\left(K_2+K_3\right)}\vec{m}+\vec{\omega}^{an}+\vec{\omega}_0(t) \qquad (17.16)$$

Note that you can imagine in this case that the torsor of moments $\vec{m}$ derives from a virtual state equation. This results in a virtual free energy density of elastic rotation per lattice site in the form

$$m_k=n\frac{\partial f^{él}_{rotation}}{\partial\omega^{él}_k}=4\left(K_2+K_3\right)\vec{\omega}^{él}\;\Rightarrow\; f^{él}_{rotation}=\frac{2\left(K_2+K_3\right)}{n}\left(\vec{\omega}^{él}\right)^2 \qquad (17.17)$$

so that the *volume density of virtual free energy of elastic rotation*, linked to the deformations by shear strains and pure elastic rotations, without volume expansion can be written

$$F^{él}_{rotation}\left(\vec{\omega}^{él}\right)=2\left(K_2+K_3\right)\left(\vec{\omega}^{él}\right)^2 \qquad (17.18)$$

The equations needed for the description of elastic shear and rotation of the cosmic lattice have yet to incorporate the topological equations for the elastic rotation vector $\vec{\omega}^{él}$, i.e. the geometro-kinetic equation and the equation of geometro-compatibility in the presence of dislocation charges

$$\vec{J}=-\frac{1}{2}\sum_k\vec{e}_k\wedge\vec{J}_k=-\frac{d\vec{\omega}^{él}}{dt}+\frac{1}{2}\overrightarrow{\mathrm{rot}}\,\vec{\phi}^{rot} \quad\text{and}\quad \lambda=\frac{1}{2}\sum_k\vec{e}_k\vec{\lambda}_k=\mathrm{div}\,\vec{\omega}^{él} \qquad (17.19)$$

With regards to density ρ of inertial mass of lattice, the hypothesis 2 and 3 allow to insure it is a constant

$$\rho=m\left(n+n_I-n_L\right)=mn\left(1+C_I-C_L\right)=cste \qquad (17.20)$$

so that the evolution equation of this density in the local referential $Ox_1x_2x_3$ allow us to deduce that the divergence of $n\vec{p}^{\,rot}$ is null

$$\frac{\partial\rho}{\partial t}=0=-\mathrm{div}\left(n\vec{p}^{\,rot}\right)\quad\Rightarrow\quad\mathrm{div}\left(n\vec{p}^{\,rot}\right)=0 \qquad (17.21)$$

This quantity $n\vec{p}^{\,rot}$ is directly deduced from *(17.9)* and can be written under the following form

$$n\vec{p}^{\,rot}=mn\left[\vec{\phi}^{rot}+\left(C_I-C_L\right)\vec{\phi}^{rot}+\frac{1}{n}\left(\vec{J}^{rot}_I-\vec{J}^{rot}_L\right)\right]=\rho\vec{\phi}^{rot}+m\left(\vec{J}^{rot}_I-\vec{J}^{rot}_L\right) \qquad (17.22)$$

.

From relations *(8.22)* and *(10.28)*, we can suppose that there are no sources of charges of rotation S^{λ} in the lattice

Hypothesis 4: $$S^{\lambda} = \left[\frac{d}{dt}\left(\operatorname{div}\vec{\omega}^{él}\right) - \operatorname{div}\left(\frac{d\vec{\omega}^{él}}{dt}\right)\right] \equiv 0 \tag{17.23}$$

so that the equation of continuity for the charges of rotation can be written

$$\frac{d\lambda}{dt} = -\operatorname{div}\vec{J} \tag{17.24}$$

Finally, it is still possible to establish an *energetic balance equation* from the equations *(17.15)* and *(17.9)*

$$-\vec{m}\vec{J} = \vec{m}\frac{d\vec{\omega}^{él}}{dt} - \vec{\phi}^{rot}\frac{d(n\vec{p}^{rot})}{dt} - \operatorname{div}\left(\frac{1}{2}\vec{\phi}^{rot} \wedge \vec{m}\right) \tag{17.25}$$

The relations thus obtained for the cosmic lattice in the local coordinates $\boldsymbol{O}x_1x_2x_3$ of ***GO***, in translation $\vec{\phi}_O(t)$ and in rotation $\dot{\vec{\omega}}_O(t)$ in the absolute referential, are reported in table 17.1, where they are compared with the *Maxwell's equations of electromagnetism in an electrically charged environment which is conductive, magnetic and dielectric.*

There is a *very strong analogy* between these two sets of equations, except that the evolution equations involve the total (material) derivative, while Maxwell's equations involve the partial derivative with respect to time. However, it must be remembered that the total derivative *(2.20)* in the local frame can be replaced by the partial derivative with respect to time if the strains are small enough and / or slow enough close to the origin of the local frame, which we did in table 17.1!

On the analogy with the Maxwell's equations of Electro-Magnetism

The analogy between the cosmological equations of a lattice taken at almost constant and homogeneous volume expansion and Maxwell's equations of electromagnetism is entirely remarkable, because it is absolutely complete, as clearly shown in the equations given in table 12.1 and 17.1. In fact, our equations contain an additional density of "rotational" flexion charges in the second pair of equations, which has no counterpart in the Maxwell's equations. By then assuming a cosmological lattice in which $\vec{\lambda}^{rot}$ can be neglected

Hypothesis 5: $$\vec{\lambda}^{rot} \approx 0 \tag{17.26}$$

the analogy between the equations of the cosmic lattice and the equations of Maxwell becomes absolutely exact, and deserve further comments.

On the analogy between the charges of rotation and the electrical charges

The equations of table 17.1 show a complete analogy between the density λ of charges of rotation and the density ρ of electrical charges, as well as the vectorial flow $\vec{J}$ of charges of rotation and the density of electrical current $\vec{j}$.

Table 17.1 - "Maxwellian" formulation of the equations of evolution of a cosmic lattice in the local framework $Ox_1x_2x_3$ of GO

$\begin{cases} -\dfrac{\partial(2\vec{\omega}^{él})}{\partial t}+\overrightarrow{\mathrm{rot}}\,\vec{\phi}^{rot}\cong(2\vec{J}) \\ \mathrm{div}\,(2\vec{\omega}^{él})=(2\lambda) \end{cases}$	$\Leftrightarrow$	$\begin{cases} -\dfrac{\partial\vec{D}}{\partial t}+\overrightarrow{\mathrm{rot}}\,\vec{H}=\vec{j} \\ \mathrm{div}\,\vec{D}=\rho \end{cases}$
$\begin{cases} \dfrac{\partial(n\vec{p}^{rot})}{\partial t}\cong-\overrightarrow{\mathrm{rot}}\left(\dfrac{\vec{m}}{2}\right)+2K_2\vec{\lambda}^{rot} \\ \mathrm{div}(n\vec{p}^{rot})=0 \end{cases}$	$\Leftrightarrow$	$\begin{cases} \dfrac{\partial\vec{B}}{\partial t}=-\overrightarrow{\mathrm{rot}}\,\vec{E} \\ \mathrm{div}\,\vec{B}=0 \end{cases}$
$\begin{cases} (2\vec{\omega}^{él})=\dfrac{1}{(K_2+K_3)}\left(\dfrac{\vec{m}}{2}\right)+(2\vec{\omega}^{an})+\left(2\vec{\omega}_0(t)\right) \\ (n\vec{p}^{rot})=(nm)\left[\vec{\phi}^{rot}+\left(C_I-C_L\right)\vec{\phi}^{rot}+\left(\dfrac{1}{n}\left(\vec{J}_I^{rot}-\vec{J}_L^{rot}\right)\right)\right] \end{cases}$	$\Leftrightarrow$	$\begin{cases} \vec{D}=\varepsilon_0\,\vec{E}+\vec{P}+\vec{P}_0(t) \\ \vec{B}=\mu_0\left[\vec{H}+\left(\chi^{para}+\chi^{dia}\right)\vec{H}+\vec{M}\right] \end{cases}$
$\left\{ \dfrac{\partial(2\lambda)}{\partial t}\cong-\mathrm{div}(2\vec{J}) \right.$	$\Leftrightarrow$	$\left\{ \dfrac{\partial\rho}{\partial t}=-\mathrm{div}\,\vec{j} \right.$
$\begin{cases} -\left(\dfrac{\vec{m}}{2}\right)(2\vec{J})\cong \\ \vec{\phi}^{rot}\,\dfrac{\partial(n\vec{p}^{rot})}{\partial t}+\left(\dfrac{\vec{m}}{2}\right)\dfrac{\partial(2\vec{\omega}^{él})}{\partial t}-\mathrm{div}\left(\vec{\phi}^{rot}\wedge\left(\dfrac{\vec{m}}{2}\right)\right) \end{cases}$	$\Leftrightarrow$	$\begin{cases} -\vec{E}\,\vec{j}= \\ \vec{H}\,\dfrac{\partial\vec{B}}{\partial t}+\vec{E}\,\dfrac{\partial\vec{D}}{\partial t}-\mathrm{div}\left(\vec{H}\wedge\vec{E}\right) \end{cases}$
$\left\{ c_t=\sqrt{\dfrac{K_2+K_3}{mn}} \right.$	$\Leftrightarrow$	$\left\{ c=\sqrt{\dfrac{1}{\varepsilon_0\mu_0}} \right.$

On the analogy between the anelasticity of the lattice and the dielectric properties of matter

The phenomenon of anelasticity introduced here by the term $2\vec{\omega}^{an}$ becomes in comparison with Maxwell's equations of electromagnetism, analogous to the dielectric polarization in the relationship $\vec{D}=\varepsilon_0\,\vec{E}+\vec{P}+\vec{P}_0(t)$, giving the electric displacement $\vec{D}$ versus electric field $\vec{E}$ and polarization of matter $\vec{P}$.
This analogy between fields $2\vec{\omega}^{an}$ and $\vec{P}$ is very strong since the possible phenomenological behavior of these two quantities are entirely similar, as shown in the relaxation, resonant or hysteresis behaviors described in section 7.8 and figures 7.7 and 7.10. For example, in the case of a pure relaxation, it is possible to connect $\vec{\omega}$ and $\vec{m}$ by means of a complex modulus, as it is possible to connect $\vec{D}$ and $\vec{E}$ via a similar complex dielectric coefficient in electromagnetism (in fact, a deeper comparison would show that the behaviors associated with thermal activation also present analogies).

As for the term of homogeneous dielectric polarization $\vec{P}_0(t)$ we introduced here, it is the analogue of a term of global rotation of the local coordinate $Ox_1x_2x_3$ in the absolute ***GO*** referential. This term therefore disappears in the case where the local coordinate system $Ox_1x_2x_3$ is only in translation $\vec{\phi}_O(t)$ relative to the absolute referential.

On the analogy between mass transport in the lattice and magnetism of matter

As $n\vec{p}^{rot}$ represents both the average quantity of movement per unit volume of the solid and the average mass flow within the solid, we deduce that the mass flow within the solid is due at the same time to a transport of mass $nm\vec{\phi}^{rot}$ with velocity $\vec{\phi}^{rot}$ corresponding to the movement of the lattice, second to a mass transport $nm\left(C_I - C_L\right)\vec{\phi}^{rot}$ at velocity $\vec{\phi}^{rot}$ by the driving movement of the point defects by the lattice and finally to a mass transport $m\left(\vec{J}_I^{rot} - \vec{J}_L^{rot}\right)$ due to the phenomenon of self-diffusion of vacancies and interstitials.

Each of these mass transports has an analog in Maxwell's equations of electromagnetism. The mass transport $nm\vec{\phi}^{rot}$ by the lattice is analogous to the term $\mu_0\vec{H}$ of the *magnetic induction* in a vacuum. The mass transport $nm\left(C_I - C_L\right)\vec{\phi}^{rot}$ by dragging along the point defects by the lattice perfectly corresponds to the term $\mu_0(\chi^{para} + \chi^{dia})\vec{H}$ of magnetism, wherein the *magnetic susceptibility* is composed of two parts: the *positive paramagnetic susceptibility* χ^{para}, which becomes the analog of the concentration C_I of interstitials, and the *negative diamagnetic susceptibility* χ^{dia}, which is therefore analogous to the concentration of vacancies C_L.

With regards to the phenomena of auto-diffusion by the holes and interstitials, we have in these equations the term $m\left(\vec{J}_I^{rot} - \vec{J}_L^{rot}\right)$ which links the last part of $n\vec{p}^{rot}$ to velocities $\Delta\vec{\varphi}_L^{rot}$ and $\Delta\vec{\varphi}_I^{rot}$ of auto-diffusion of point defects

$$n\vec{p}^{rot}_{auto-diffusion} = m\left(\vec{J}_I^{rot} - \vec{J}_L^{rot}\right) = mn\left(C_I\Delta\vec{\varphi}_I^{rot} - C_L\Delta\vec{\varphi}_L^{rot}\right) \tag{17.27}$$

As an example we can imagine a hypothetical lattice in which the vacancies are tightly anchored to the lattice ($B_L(\tau,T) \to \infty$), while the interstitials are free to move ($B_I(\tau,T) \to 0$). The equations of movement *(7.61)* then become, by taking into account hypothesis 2

$$\begin{cases} \Delta\vec{\varphi}_L^{rot} \cong \dfrac{m}{B_L}\dfrac{\partial\vec{\phi}^{rot}}{\partial t} \\ m\dfrac{\partial\Delta\vec{\varphi}_I^{rot}}{\partial t} \cong -m\dfrac{\partial\vec{\phi}^{rot}}{\partial t} \end{cases} \tag{17.28}$$

The solutions to these hypothetical equations are then simply written, by introducing a constant velocity vector $\vec{v}_I^{rot}$

$$\begin{cases} \Delta\vec{\varphi}_L^{rot} \to 0 \\ \Delta\vec{\varphi}_I^{rot} \cong \vec{\phi}^{rot} + \vec{v}_I^{rot} \end{cases} \tag{17.29}$$

As a consequence, the quantity of movement $n\vec{p}^{rot}$ within the lattice can be written

$$n\vec{p}^{rot} = nm\left[\vec{\phi}^{rot} + \left(C_I - C_L\right)\vec{\phi}^{rot} + C_I\Delta\vec{\varphi}_I^{rot}\right] = nm\left[\vec{\phi}^{rot} + \left(2C_I - C_L\right)\vec{\phi}^{rot} + C_I\vec{v}_I^{rot}\right] \tag{17.30}$$

Mass transport $n\vec{p}^{rot}$ now has a term $\left(2C_I - C_L\right)\vec{\phi}^{rot}$ associated with both vacancies and interstitials, whose coefficient $\left(2C_I - C_L\right)$ is analogous to the magnetic susceptibility χ in electromagnetism, and that can take a positive or negative value depending on concentrations C_I and

C_L of point defects. It further contains the term $nmC_I\vec{v}_I^{rot}$ associated with mass transport by inertial conservative interstitial movement, which is perfectly analogous to the permanent magnetization $\vec{M}$ of the ferromagnetic and antiferromagnetic materials in electromagnetism.
The presence of the constant term $nmC_I\vec{v}_I^{rot}$ in $n\vec{p}^{rot}$ clearly corresponds to a non-Markovian type of process, since the value must depend on the history of this hypothetical solid lattice. One could imagine for instance that the movement of interstitials is controlled by a dry type of friction with the lattice, in which case there would be a critical force of depinning for interstitials, which would lead to the emergence of cycles of hysteresis of $\Delta\vec{\varphi}_I^{rot}(t)$ as a function of $\vec{\phi}^{rot}(t)$. This is absolutely similar to the cycles of hysteresis of magnetization $\vec{M}(t)$ as a function of the magnetic field $\vec{H}(t)$ observed in ferromagnetic or antiferromagnetic materials!

On the complete analogy with the electromagnetism theory

The complete analogy between the parameters of our theory and the Maxwell's theory of electromagnetism is reported in table 17.2

Table 17.2 - The complete analogy with the Maxwell's theory of electromagnetism

$\vec{\omega}^{él}$	$\Leftrightarrow$	$\vec{D}$ = *electric field of displacement*
$n\vec{p}^{rot}$		$\vec{B}$ = *magnetic induction field*
$\vec{m}/2$		$\vec{E}$ = *electric field*
$\vec{\phi}^{rot}$		$\vec{H}$ = *magnetic field*
$2\vec{J}$	$\Leftrightarrow$	$\vec{j}$ = *electric current*
2λ		ρ = *density of electric charges*
$\vec{\lambda}^{rot}$		? = *unknown*
$1/(K_2+K_3)$	$\Leftrightarrow$	ε_0 = *dielectric permittivity of vacuum*
nm		μ_0 = *magnetic permeability of vacuum*
$2\vec{\omega}^{an}$	$\Leftrightarrow$	$\vec{P}$ = *dielectric polarization of matter*
$(C_I - C_L)$		$(\chi^{para}+\chi^{dia})$ = *paramagnetic and diamagnetic susceptibility of matter*
$(\vec{J}_I^{rot} - \vec{J}_L^{rot})/n$		$\vec{M}$ = *magnetization of matter*
$\vec{\phi}^{rot} \wedge \vec{m}/2$	$\Leftrightarrow$	$\vec{H} \wedge \vec{E}$ = *vector of Poynting*
$c_t = \sqrt{(K_2+K_3)/mn}$		$c = \sqrt{1/(\varepsilon_0\mu_0)}$ = *speed of light*

On the effects of volume expansion of the lattice in the absolute frame of the GO

In this analogy, the existence of a uniform nonzero translation $\vec{\phi}_O(t)$ of the lattice, equivalent to a translation of the local coordinate system $Ox_1x_2x_3$ relative to the absolute referential $Q\xi_1\xi_2\xi_3$ of the **GO** would be analogous in the Maxwell equations to a homogeneous magnetic field $\vec{H}_0(t)$ in space. This last remark implies that if a solid lattice was expanding in the absolute referential frame of ***GO***, there should appear a field $\vec{\phi}_O(t)$ in the local referential frame $\boldsymbol{O}x_1x_2x_3$. This field $\vec{\phi}_O(t)$ should be similar to *a locally homogeneous magnetic field* $\vec{H}_0(t)$ *in space if the universe was expanding*, and which should point in the direction of movement of the local coordinate of the observer relative to absolute space!

On the non-existence of magnetic monopoles in this analogy

The equation $\operatorname{div}(n\vec{p}^{rot})=0$ reflects the fact that we consider a solid with a homogeneous field of static volume expansion. The existence of a non-null and constant value of $\operatorname{div}(n\vec{p}^{rot})$ such that

$$\operatorname{div}(n\vec{p}^{rot})=\operatorname{div}\left[mn\left(1+C_I-C_L\right)\vec{\phi}^{rot}\right]+\operatorname{div}\left[m\left(\vec{J}_I^{rot}-\vec{J}_L^{rot}\right)\right]\neq 0 \tag{17.31}$$

would imply that there exists a constant and divergent field of velocity $\vec{\phi}^{rot}$ of the sites of the lattice, and thus, with hypothesis $\tau = cste$, a non-zero source of sites of lattice S_n

$$\operatorname{div}\vec{\phi}^{rot}\cong\underbrace{\partial\tau/\partial t}_{=0}+\frac{S_n}{n}=\frac{S_n}{n} \tag{17.32}$$

or that we have a constant and divergent flow of auto-diffusion $m\left(\vec{J}_I^{rot}-\vec{J}_L^{rot}\right)$, and as a consequence, localized and non null sources of point defects S_{I-L}, S_L^{pl} and/or S_I^{pl}, which would be written, by taking into account the hypothesis that $C_I = cste$ and $C_L = cste$, as

$$\begin{cases} n\dfrac{\partial C_L}{\partial t}\cong 0 \qquad \Rightarrow \qquad \operatorname{div}\vec{J}_L^{rot}=\left(S_{I-L}+S_L{}^{pl}\right)-C_L\left(S_L{}^{pl}-S_I{}^{pl}\right) \\ \\ n\dfrac{\partial C_I}{\partial t}\cong 0 \qquad \Rightarrow \qquad \operatorname{div}\vec{J}_I^{rot}=\left(S_{I-L}+S_I{}^{pl}\right)-C_I\left(S_L{}^{pl}-S_I{}^{pl}\right) \end{cases} \tag{17.33}$$

As part of the analogy with electromagnetism, a relationship $\operatorname{div}(n\vec{p}^{rot})=cste\neq 0$ would be like a $\operatorname{div}\vec{B}=cste\neq 0$ relationship. Now this last relationship shows the *well-known concept of magnetic monopoles, particles of unipolar magnetic charges*, suggested by some theories, but never observed experimentally, and who would therefore be localized and continuous source of lattice sites or of point defects in the lattice!

In fact, the existence of similarity between two theories is always a very fruitful and successful thing in physics by the reciprocal contribution of one theory to the other. In our case, it is clear that this analogy with the electromagnetic field theory will enable us subsequently to use the whole arsenal of theoretical tools developed for a long time in field theory, such as for example, the Lorentz transformation or delayed potential theory. In the other direction, the theory developed here is actually a much more complex theory that classical electromagnetism, since it stems from a tensorial theory, which can be reduced to a vectorial theory by contraction of tensor indices. We can also choose more specific cases with less restrictive hypothesis in the solid lattice. Considering the tensorial aspect of solid lattice theory and by relaxing the more restrictive hypothesis, the analogy will become particularly interesting and fruitful, as we shall see later.

On the possible existence of "vectorial electrical charges" in this analogy

One can legitimately ask what could be the analogy of the density of flexion charges $\vec{\lambda}^{rot}$ in the Maxwell's equations. If there were a quantity $\vec{\lambda}^{rot}$ similar in the Maxwell equations, one could hypothetically call it a density $\vec{\rho}$ of "*vectorial electric charges*" by postulating the following analogy

$$\vec{\rho} \quad \Leftrightarrow \quad \vec{\lambda}^{rot} \tag{17.34}$$

The equations of Maxwell would then be written a little differently from the known equations, with an extra term of charge but not in the equation $\operatorname{div}\vec{B}=0$ as suggested in the theories of magnetic monopoles, but in the equation $\partial\vec{B}/\partial t=-\overrightarrow{\operatorname{rot}}\,\vec{E}$ *,* in the following way

$$\begin{cases} -\dfrac{\partial\vec{D}}{\partial t}+\overrightarrow{\operatorname{rot}}\,\vec{H}=\vec{j} \\ \operatorname{div}\vec{D}=\rho \end{cases} \quad \text{and} \quad \begin{cases} \dfrac{\partial\vec{B}}{\partial t}=-\overrightarrow{\operatorname{rot}}\,\vec{E}+\kappa\vec{\rho} \\ \operatorname{div}\vec{B}=0 \end{cases} \tag{17.35}$$

in which κ is *a new electric coefficient*, analogous to the modulus $2K_2$

$$\kappa \Leftrightarrow 2K_2 \tag{17.36}$$

In the static case, if such a vectorial charge did in fact exist, the equation containing it would be written as

$$\frac{\partial\vec{B}}{\partial t}=-\overrightarrow{\operatorname{rot}}\,\vec{E}+\kappa\vec{\rho}=0 \quad\Rightarrow\quad \overrightarrow{\operatorname{rot}}\,\vec{E}=\kappa\vec{\rho} \quad\Rightarrow\quad \overrightarrow{\operatorname{rot}}\,\vec{D}=\varepsilon_0\kappa\vec{\rho} \tag{17.37}$$

so that the density $\vec{\rho}$ of *«vectorial electric charges»* would be the source of a rotational electric field $\vec{E}$ and a rotational electric field of displacement $\vec{D}$, just as the scalar density ρ of electrical charges is the source of a divergent electric field of displacement $\vec{D}$

$$\begin{cases} \operatorname{div}\vec{D}=\rho \\ \overrightarrow{\operatorname{rot}}\,\vec{D}=\varepsilon_0\kappa\vec{\rho} \end{cases} \tag{17.38}$$

If we now compare the coefficients of both theories we obtain the following analogies

$$\varepsilon_0 \Leftrightarrow \frac{1}{(K_2+K_3)} \quad \textit{et} \quad \kappa \Leftrightarrow 2K_2 \quad\Rightarrow\quad \varepsilon_0\kappa \Leftrightarrow \frac{2K_2}{K_2+K_3} \tag{17.39}$$

However the experimental observations have never shown the existence of such "*vectorial electric charges".* Indeed, two reasons can be invoked to explain this state of affairs: either the "*vectorial electric charges*" simply do NOT exist, or the coefficient $\varepsilon_0\kappa$ is so small that we do not observe the presence of these *«vectorial electric charges»*, so

$$|\varepsilon_0\kappa| << 1 \quad\Leftrightarrow\quad \left|\frac{2K_2}{K_2+K_3}\right| << 1 \tag{17.40}$$

Starting from the saying that "*everything which is not prohibited must exist"*, we can here deduce a new conjecture for our theory

Conjecture 4: *the modules K_2 and K_3 must satisfy the relation $|K_2|<<|K_3|$,*
<u>or else</u> the module K_2 could be null $(K_2=0)$ (17.41)

We will revisit this conjecture later as it is going to play a major role.