Chapter 3

The «Cosmological Lattice» and its properties

By the introduction of an elastic strain energy of the lattice, called the *elastic internal energy*, expressed *per unit volume of the lattice* and dependent at the same time on *the volume expansion, the shear strains and especially on the local rotations within the lattice*, we obtain an imaginary lattice, which we will call *"cosmological lattice"*. This lattice has a very particular *Newton's equation*, in which appears in particular *a new term of force*, which is directly related to the energy of distortion due to the topological singularities contained in the lattice, and which will be called thereafter to play a fundamental role in analogies with Gravitation and with Quantum Physics.

The propagation of waves in the cosmological lattice presents very interesting features. The propagation of transverse waves with linear polarization is always associated with longitudinal wavelets, and the propagation of pure transverse waves can only be done by *waves with circular polarization*, which will then have a direct link with the existence of photons. On the other hand, the propagation of longitudinal waves can disappear in favor of the appearance of *localized modes of longitudinal vibration* in the case where the volume expansion of the lattice is less than a certain critical value, which will subsequently have a direct link with quantum physics.

A curvature of the wave rays also appears in the vicinity of a singularity of the volume expansion of the lattice. This phenomenon makes it possible to find the conditions which the expansion field associated with a topological singularity must satisfy so that a trap appears which captures all the transverse waves, in other words a *"black hole»*.

A finite cosmological lattice in absolute space can exhibit *a dynamic volume expansion and / or contraction*, provided that it contains a certain amount of kinetic energy, a phenomenon quite similar to the cosmological expansion of the Universe. According to the signs and the values of the elastic modules, several types of cosmological behaviors of the lattice are possible, some of which present the phenomena of *big bang, rapid inflation* and *acceleration of the speed of expansion*, and which can be followed in some cases of a *re-contraction of the lattice* leading to a *big-bounce* phenomenon. We deduce that it is the elastic energy contained in the lattice and due to the expansion which is responsible for these phenomena, and in particular for *the increase in the speed of expansion*, phenomenon which is observed on the current Universe by astrophysicists and which is attributed by them to a hypothetical *"dark energy"*.

The «cosmological lattice» and its Newton's equation

Let us introduce an imaginary solid lattice, which will be arbitrarily called *"cosmological lattice"* and whose *internal energy of distortion is expressed per volume unit* in the form of a development of the volume expansion τ , the elastic and anelastic shears $\vec{\alpha}_i^{\,él}$ and $\vec{\alpha}_i^{\,an}$, but also directly from the vectors of elastic and anelastic rotation $\vec{\omega}^{\,él}$ and $\vec{\omega}^{\,an}$. Our initial

conjecture is therefore *a priori* the following: the internal energy of the "cosmological lattcie" is expressed as a function $U^{déf} = U^{déf}\left[\tau, \tau^2, (\vec{\alpha}_i^{\,él})^2, (\vec{\omega}^{\,él})^2, (\vec{\alpha}_i^{\,an})^2, (\vec{\omega}^{\,an})^2\right]$ of the state of distortion per unit volume of the lattice.

Such a lattice in fact corresponds to the most general isotropic perfect lattice that one can imagine if one makes his energy depend both linearly on the volume expansions and quadratically on the volume expansions, shear strains and rotations by torsional deformation. Still in the spirit of simplification, we can also assume that there is no elasticity by volume expansion in this lattcie. The state function per unit volume of this cosmological lattice is therefore written in the form of the expression presented in figure 3.1, in which appears four elastic constants K_0, K_1, K_2, K_3 and two anelastic constants K_1^{an}, K_2^{an} which completely characterize the elasticity and the anelasticity of this lattice.

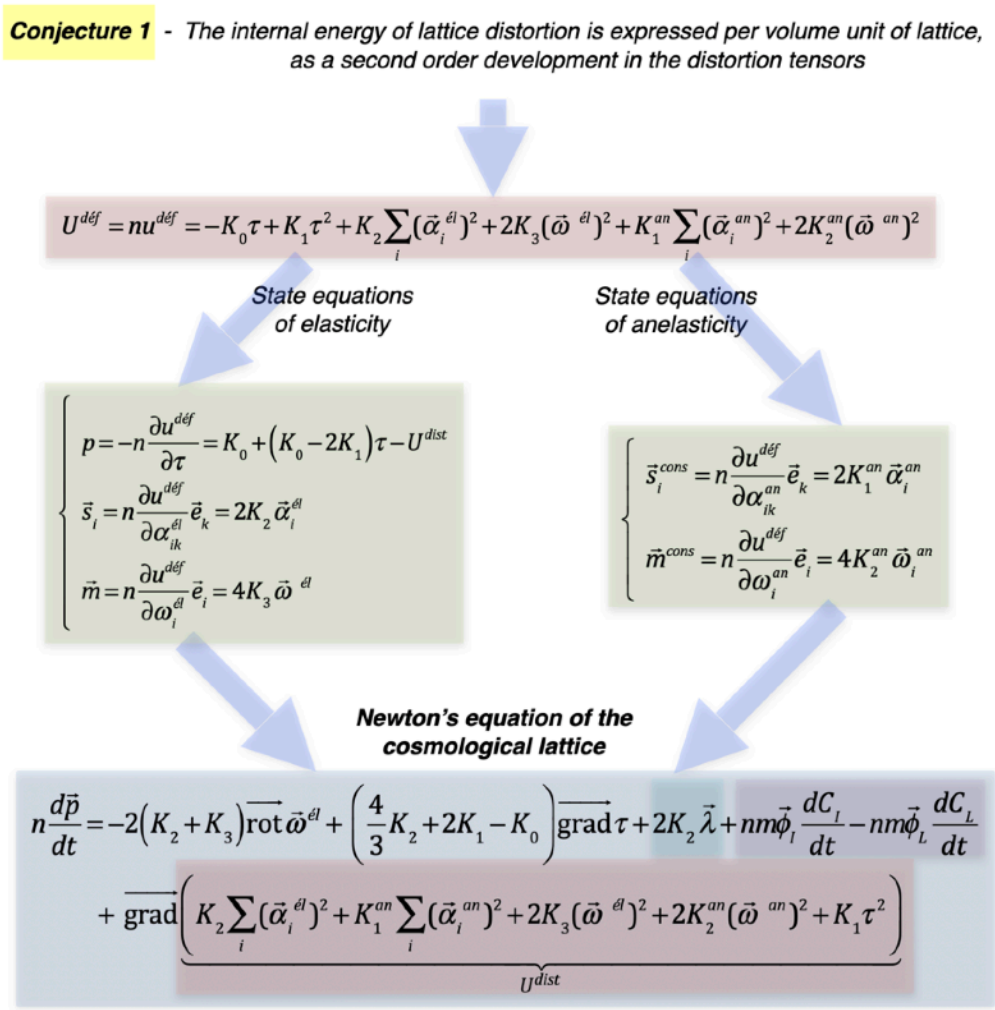

Figure 3.1 *- The first conjecture which leads to the cosmological lattice and its Newton's equation*

One then deduces from it five *equations of state of elasticity and anelasticity,* respectively for the scalar of pressure p, the symmetrical transverse tensors of shear stresses $\vec{s}_i$ and $\vec{s}_i^{\,cons}$ as well as the moment vectors $\vec{m}$ and $\vec{m}^{cons}$, which are also reported in the figure 3.1. These state equations can be used to carry out a rather tedious computation which finally provides *the*

equation of Newton of the cosmological lattcie reported in figure 3.1. This equation is indeed a Newton's equation because it provides *the temporal variation of the amount of momentum per lattice site*, taken along the trajectory of the lattice, according to the spatial variations of the tensors of distortion, via the elastic and anelastic constants.

We will see later that this Newton's equation will play an absolutely central role on the behavior of the cosmological lattice. It is quite complicated, in particular by the presence of *the density* $\vec{\lambda}$ *of flexion charge*, of terms related to *the diffusion of intrinsic point defects* and especially of *the new term of force* depending directly on *the volume density of internal distorsion energy* U^{dist}, which is directly related to the energy of distortion, namely the quadratic terms of the deformation energy $U^{déf}$, due to the topological singularities contained in the lattice, and which will be called thereafter to play a fundamental role in the analogies with Gravitation and with Quantum Physics.

The fact of introducing a rotational energy with terms in $\vec{\omega}^{él}$ and $\vec{\omega}^{an}$ into the expression of the internal energy of the cosmological lattice, as well as the fact of developing its internal energy per unit of volume and not per site of lattice, are not not at all elementary to understand, and really make this lattice *a perfectly imaginary lattice* of which there is absolutely no equivalent among the usual solids.

Rather than embarking now on a superfluous search for interpretation of the *"hidden face"* of this imaginary lattice, it seems preferable to start by exploring in detail the consequences that this hidden face implies in terms of the behaviors that the cosmological lattice can present. It is therefore to this task that devolves the rest of this book, which will be devoted to extracting the substantial core from this hidden face. To do this, we will show that it is *the Newton's equation* that we have just deduced which has spectacular properties and which is at the heart of the very many analogies that we will develop in the following with the great theories of physics, namely Maxwell's Electromagnetism, Lorentz Transformation, Einstein's Special Relativity, Newton's Gravitation, Einstein's General Relativity, and even Quantum Physics and the Standard Model of Elementary Particles.

Transverse and longitudinal perturbations in the cosmological lattice

We will first be interested in the transverse and longitudinal perturbations that can reign within the cosmological lattice. There are quite surprising vibrational phenomena (figure 3.2), such as t*he existence of a mode of propagation of linearly polarized transverse waves which are necessarily coupled to longitudinal wavelets*, while *circularly polarized transverse waves are free of these wavelets*. There is also the possibility of propagation of *longitudinal waves*. But, under certain conditions which strongly depend on the elastic moduli and the state of expansion of rest of the lattice, the mode of longitudinal propagation can disappear and be replaced by *a very astonishing mode of localized vibrations of expansion*, which will play by thereafter a key role in analogies with gravitation and quantum physics.

To discuss these various modes of mechanical perturbations in the cosmological lattice, we will make simplifying hypotheses, namely that there are no anelastic perturbations, therefore that $\vec{\alpha}_i^{an} = 0$ and $\vec{\omega}^{an} = 0$, that there are no vacancies and self -interstitials, therefore that $C_I = C_L \cong 0$ and that there is no density $\vec{\lambda}$ of flexion charges, therefore that $\vec{\lambda} = 0$.

We will further assume that the state of the background volume expansion of the lattice is a constant τ_0, so that the total volume expansion of the lattice is written $\tau = \tau_0 + \tau^{(p)}$, in which $\tau^{(p)}$ represents the mechanical perturbations of the expansion around the rest value τ_0. To calculate the various perturbation modes, we inject the expression of the momentum of the perturbations, that is $\vec{p}^{(p)} = m\dot{\vec{\phi}}^{(p)}$, into Newton's equation in figure 3.1. We can then describe four different perturbations modes:

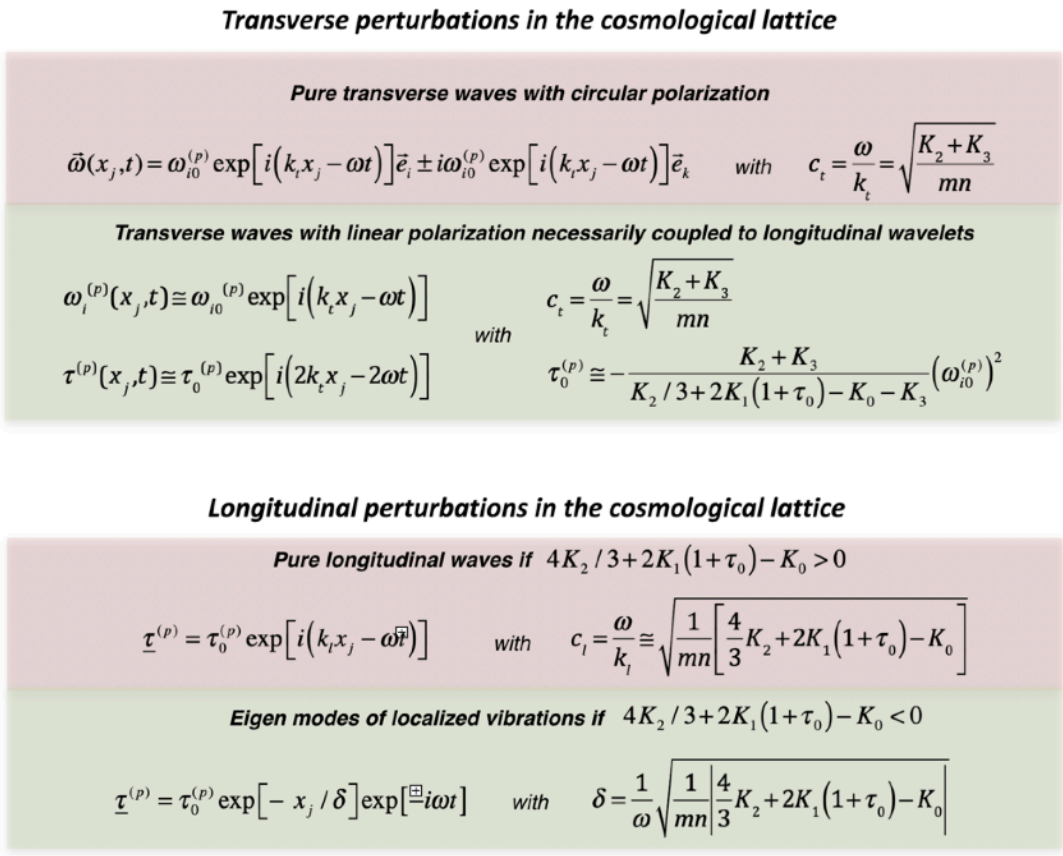

Figure 3.2 *- Transverse and longitudinal perturbations in the cosmological lattice*

- *the propagation of a linearly polarized transverse wave* in the cosmological lattice, that is to say of a wave for which the vibration of the particles of the lattice is perpendicular to the direction of propagation of the wave. Such a wave satisfies a completely conventional wave equation provided that its amplitude is not too strong. But it must be accompanied by *a longitudinal wavelet* which propagates in the same direction and at the same velocity as that of transverse perturbation. The frequency of this longitudinal wavelet is twice the frequency of the transverse perturbations, and its amplitude is proportional to the square of the amplitude of the transverse perturbations. It will also be noted that the speed of propagation of the transverse perturbations strongly depends on the background volume expansion τ_0 of the lattice since the value of the lattice site density n which appears in the expressions of the speed of the

transverse waves is an exponential function $n = n_0 e^{-\tau_0}$ of the background volume expansion τ_0 of the lattice.

- *the propagation of a circularly polarized transverse wave* in the cosmological lattice, that is to say of a wave for which the vibration of the particles of the network rotates around the direction of propagation of the wave. It is said that such a wave has *a positive or negative helicity* depending on whether the direction of vibration of the particles rotates dextrorotatory or levorotatory at an angle of 360 ° over one wavelength. Circularly polarized transverse waves are pure, in the sense that they are not coupled to longitudinal wavelets.

- *the propagation of a longitudinal wave* in the cosmological lattice, that is to say of a wave for which the direction of vibration of the particles of the lattice is the same as the direction of propagation of the wave is quite complicated to compute, because *the propagation equations are strongly non-linear* and depend directly on the state of rest expansion τ_0 of the lattice. In this case, only very small perturbations $\tau^{(p)}$ within the lattice are considered, which makes it possible to linearize the wave equations. The celerity c_l of longitudinal waves of small amplitude which appears in figure 3.2 is expressed with the square root of an argument which must be positive for it to exist. As this argument depends both on the elastic constants K_0, K_1, K_2 and on the state of rest volume expansion τ_0 of the latttice, directly and exponentially via the expression $n = n_0 e^{-\tau_0}$, *the existence of a longitudinal wave propagation is subject to the condition* $4K_2/3 + 2K_1(1+\tau_0) - K_0 > 0$.

- *the longitudinal perturbations in the form of localized eigen modes of vibrations of the volume expansion*. In the cosmological lattice, if the celerity c_l of the longitudinal waves becomes an imaginary number (this is what is called the square root of a negative number), *there is no longer any propagation of longitudinal waves*. In this case, we can rewrite the complex perturbation solution in the form shown in figure 3.2. There arises here a very surprising phenomenon, namely *the appearance of localized eigen modes of longitudinal vibrations*, which do not propagate at great distance, but which are on the contrary confined on distances of the order of δ. For high amplitudes of these localized modes of longitudinal perturbations, these will become non-linear and will therefore strongly depend on the amplitude $\tau^{(p)}$ of the perturbations. The appearance of these "strange" modes is obviously subject to the condition that $4K_2/3 + 2K_1(1+\tau_0) - K_0 < 0$.

It is remarkable that in the cosmological lattice, the speed $c_t = \omega / k_t$ of the transverse waves depend exponentially on the state of expansion τ of the lattice via the value of a multiplicative term being worth $e^{\tau/2}/\sqrt{n_0}$. The same goes for the speed c_l of the longitudinal waves, although there is also a dependence on their speed through a term τ within the argument of the root.

It is also remarkable that the linearly polarized transverse waves are necessarily coupled to longitudinal wavelets in the cosmological lattice, and that the only transverse waves which are pure, not coupled to longitudinal wavelets, are then the transverse waves with right or left circular polarization, that is to say transverse waves of positive or negative helicity. Strangely, there is already a property specific to photons in the real universe, namely that photons are necessarily of non-zero helicity. As photons are quantum objects, we find here an astonishing peculiarity to which we will return.

The existence of domains of volume expansion of the cosmological lattice in which the

propagation of longitudinal waves is not possible, when $4K_2/3+2K_1(1+\tau_0)-K_0<0$, corresponds well by analogy with the fact that there is no propagation of longitudinal waves in thge theory of Einstein's General Relativity. Indeed, in the latter, the gravitational waves are transverse waves, defined as the propagation of perturbations of the space-time metric. These waves have a tensorial symmetry, with two independent polarizations perpendicular to the direction of propagation, contrary to the longitudinal perturbations which have a scalar symmetry.

The condition $4K_2/3+2K_1(1+\tau_0)-K_0<0$ that there are no longitudinal waves implies the existence of *a critical background expansion* τ_{0cr} *of the lattice*, the limit between the expansion domains where it exists and where there are no longitudinal waves. In order for the cosmological lattice to present analogies with Einstein's General Gravitation, with electromagnetism and with the photons of quantum physics, it is necessary that there are no longitudinal waves, but that there are pure transverse waves of circular polarization. Hence the need to make a second conjecture which is shown in figure 3.3.

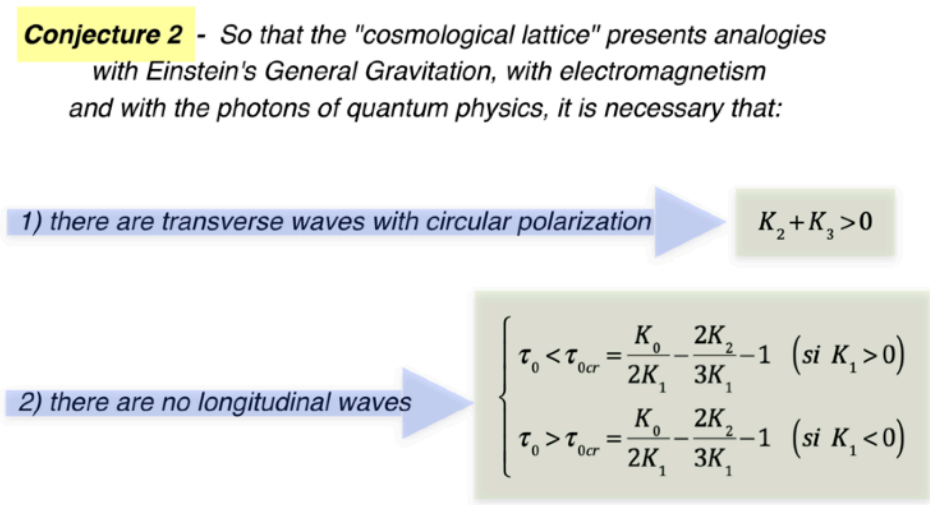

Figure 3.3 *- The second conjecture necessary for the cosmological lattice*

This conjecture implies, so that there are no longitudinal waves, that the volume expansion τ_0 of the cosmological lattice is smaller or greater than the value of *the critical expansion* τ_{0cr} depending on whether the elastic modulus K_1 is positive or negative.

In the absence of longitudinal waves, the cosmological lattice then presents *localized eigen modes of longitudinal perturbations*, therefore local vibrations of the scalar τ of volume expansion. Such modes immediately make one think of the ideas of quantum fluctuations in gravitation on a very small scale since they affect the scalar τ which undeniably has a link with the gravitational field. But these localized perturbations of the scalar of volume expansion also make one think of the quantum fluctuations of the vacuum described by quantum physics. We can therefore, on the basis of this analogy between τ and the gravitational field, ask the following question: "is gravity that must be quantified on a very small scale, or is it gravitation,

on a very small scale, responsible for quantum physics? ” We will try to provide some answers to this topical question of the most relevant.

Curvature of the wave rays by a singularity of expansion and black holes

Among the surprising behaviors that a cosmological lattice can exhibit with regard to wave propagation, there appears *a non-dispersive curvature of the wave propagation rays by a gradient of volume expansion* due to the presence of a strong topological singularity of the expansion τ . This curvature can go as far as the formation of *"black holes"* absorbing all the waves passing in its proximity, or of impenetrable *"white holes"* repelling all the waves passing in its proximity.

The fact that the celerities of the transverse and longitudinal waves increase non-linearly with the value of the static volume expansion τ via the value of the site density $n = n_0 \, \mathrm{e}^{-\tau}$ will cause a curvature of the rays of propagation of these waves if they pass in the direct vicinity from a singularity of volume expansion within the lattice, as illustrated in figure 3.4.

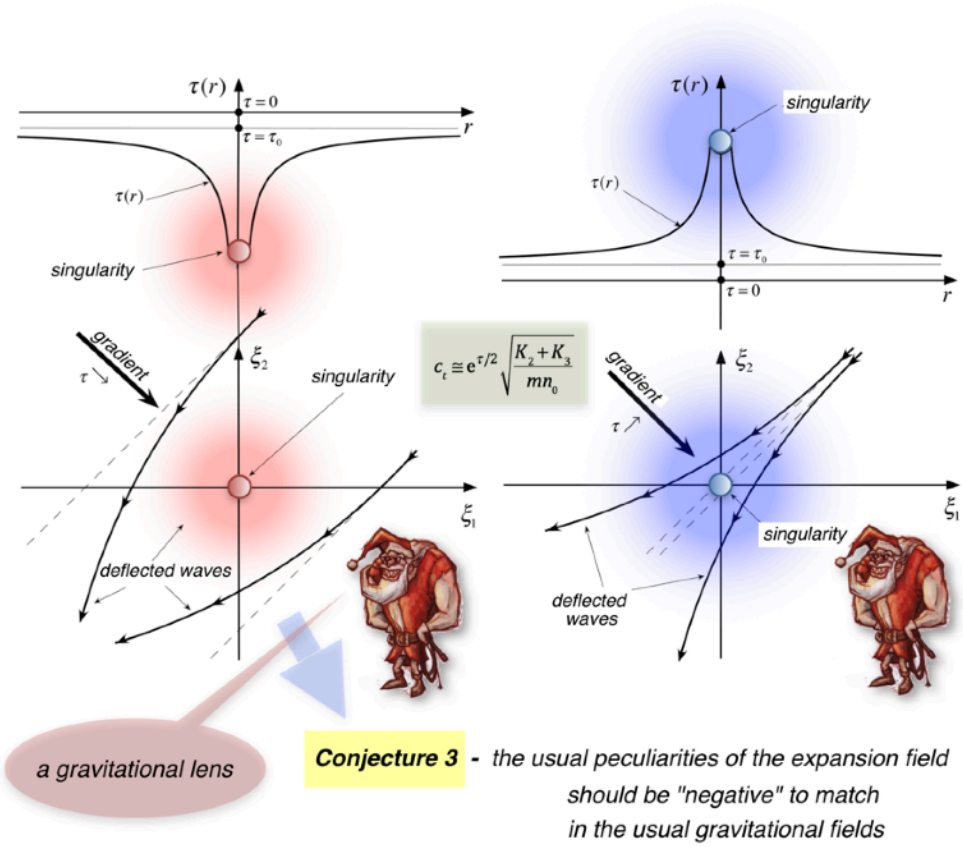

Figure 3.4 *- The curvature of the wave rays in the vicinity of a singularity of the volume expansion of spherical symmetry*

Indeed, imagine a motionless cosmological lattice in the absolute frame of reference of the ***GO*** observer and containing a singularity of volume expansion of spherical symmetry located at the center of the coordinate system $O\xi_1\xi_2\xi_3$. Let us also consider a longitudinal or transverse

wave, initially planar, arriving towards this singularity in a given initial direction. The speed of propagation increases or decreases as it approaches singularity, under the effect of lattice density $n = n_0 \, \mathrm{e}^{-\tau}$. According to whether the singularity of τ is "positive" (passes through a maximum at the origin) or "negative" (passes through a minimum at the origin), there will appear a curvature of the rays of propagation of the waves, so that the wave seems repelled by a "positive singularity" or attracted by a "negative singularity».

This phenomenon does not depend on the shape of the field around the singularity, but only on its gradient, that is to say on *the slope of the function* $\tau(r)$. For a plane wave incident on the singularity, this phenomenon of acceleration or develeration of the wave will then cause a deformation of the wave planes similar to the effect of a diverging lens in the case of a "positive singularity" or a converging lens in the case of a "negative singularity». In addition, as this phenomenon does not depend on the frequency of the wave, the singularity behaves like a converging or diverging lens of *a non-dispersive nature* in the cosmological lattice, that is to say which does not depend on the frequency of the incident wave.

Now imagine that in a motionless cosmological lattice in the absolute frame of reference of the ***GO*** observer, and containing a *"negative singularity"* of the volume expansion, of spherical symmetry, located at the center of the coordinate system, it passes a transverse wave ($c_i = c_t$) or a longitudinal wave ($c_i = c_l$). In the vicinity of the singularity, at a distance $r = r_{cr}$ from the origin of the singularity, the wave planes are always parallel to a line passing through the origin of the singularity, so that the radius of the transverse or longitudinal wave located in $r = r_{cr}$ is in fact a circle centered on the origin, as illustrated in figure 3.5. *The condition of existence* of this sphere of perturbations around the singularity effectively depends on the slope of $\tau(r)$ as explained in figure 3.5.

Thus, if a transverse or longitudinal wave passes at a distance satisfying this relationship $r \le r_{cr}$, it becomes impossible for it to escape from the virtual sphere of radius r_{cr}. If the field of singularity has a monotonous gradient increasing from its origin, the curvature of the wave rays located inside this critical sphere will be further accentuated, so that all these waves will be definitively trapped by the singularity. By analogy with the *"sphere of photons"* surrounding a black hole in general relativity, one will call *"sphere of transverse and longitudinal perturbations"* the layer located at a distance $r = r_{cr}$ from the heart of the singularity. It is clear that the existence of such *a sphere of perturbations* is subject to the condition that it lies outside the *"object"* responsible for the negative singularity of the expansion field. If the radius of this "object" is R, we deduce that the condition $r_{cr} > R$ must also be part of the conditions of existence of a *"black hole»*.

We saw in the previous section that the propagation of longitudinal waves in the perfect lattice is subject to the condition that the expression $4K_2/3 + 2K_1(1+\tau_0) - K_0$ is positive. This condition takes the form of a condition on the background volume expansion τ_0 of the lattice, which must be greater or less than a critical value τ_{0cr} given in figure 3.3. If the propagation of longitudinal waves is possible in the lattice, then the longitudinal waves will also undergo the phenomenon of trapping at the limit $r = r_{cr}$. In the event that $K_1 > 0$, there is yet another phenomenon. Indeed, if the singularity presents a monotonic gradient increasing from its origin, there could exist a radius $r = r_{cr2}^{(l)} < r_{cr}$ surrounding it beyond which the value of $\tau(\vec{r})$ becomes less than τ_{0cr}. In this case, any longitudinal wave initially trapped at the limit $r = r_{cr}$

will then reach this second limit $r = r_{cr2}^{(l)} < r_{cr}$ beyond which it will not even be able to propagate, but will increase natural modes of longitudinal vibrations located inside this volume. In the case where $K_1 < 0$, this same phenomenon does not exist since the existence of a propagation implies that $\tau(r) < \tau_0 < \tau_{0cr}$ in this case.

The cosmological lattice presents a very interesting analogy with the theory of General Gravitation of Einstein since one can find there, in the vicinity of singularities of the volume expansion, *spheres of perturbations* very similar to the *sphere of photons surrounding a black hole.* We therefore deduce from this *non-dispersive effect of the curvature of the rays by the gradients of the volume expansion* that *the scalar of volume expansion undoubtedly has a strong analog relationship with the gravitational field in General Relativity*.

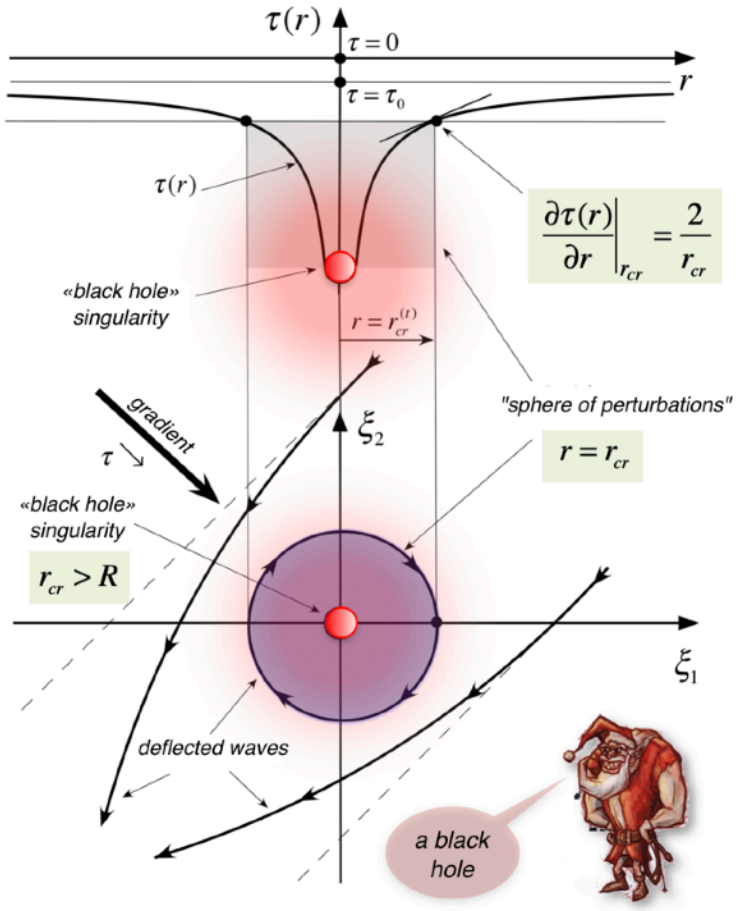

Figure 3.5 *- The "sphere of perturbations", a real "black hole" in the vicinity of a negative singularity of* τ

It is also interesting to note that only a negative singularity of τ has this property similar to that of a *"black hole"* capturing all the waves passing in its proximity, whereas a positive singularity of τ would behave like a *"white hole"* that is to say as an entity which would repel the waves, and which therefore could not be penetrated by waves. Hence the third conjecture for our analogy with Gravitation, reported in figure 3.4.

It is quite remarkable to note that the curvature of the waves by a gradient of volume expansion and the existence of a sphere of perturbations around a localized singularity of

volume expansion are exclusively due to the development of the internal energy per unit volume that we used for the cosmological lattice. Indeed, if we try to take a closer look at what would happen in the case of a perfect solid for which the internal energy would be written as a development expressed by lattice site, we would see that the speed of the transverse waves would be " invariant" whatever the state of expansion of the lattice, which obviously cannot lead to a non-dispersive curvature of the wave rays nor to the appearance of a sphere of perturbations. This analogy therefore justifies *a fortiori* the conjecture 1 that we had posed in figure 3.1, since it is this conjecture which allows the existence of the curvature of the waves and the sphere of disturbances in the cosmological lattice.

Cosmological expansion-contraction of a sphere of perfect lattice and dark energy

By considering a finite imaginary sphere of cosmological lattice, one can introduce the concept of *"cosmological evolution"* of the lattice, supposing that a certain amount of kinetic energy is injected into it. In this case, the lattice presents strong temporal variations of its volume expansion, which one can model in a very simplistic way by supposing that the volume expansion remains perfectly homogeneous in all the lattice during its evolution.

Let us imagine that, in the absolute reference frame $O\xi_1\xi_2\xi_3$, the ***GO*** observes a solid, of spherical shape, of radius R_U, made up of a lattice of cells (figure 3.6), and that this solid has a homogeneous background volume expansion which depends on time, but not on the position within the lattice, so that $\tau(t)=\tau_0(t)\neq\tau_0(\vec{\xi},t)$. In this case, the ***GO*** will observe that the radius R_U of this solid sphere depends on time $R_U = R_U(t)$ and therefore that this sphere will

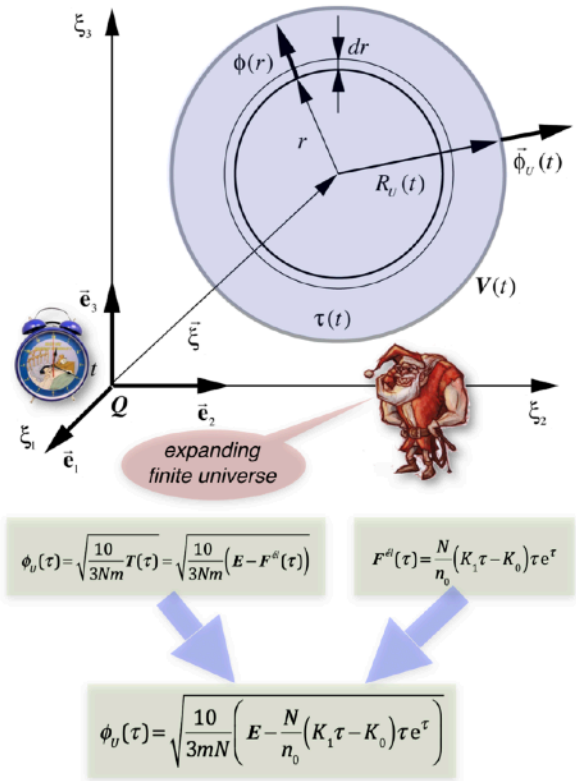

Figure 3.6 - *"Cosmological" volume expansion $\tau(t)$ of an imaginary solid sphere*

tend to expand or contract. This behavior, which one could qualify as *"cosmological"* by analogy to *the cosmological theories of expansion of the Universe*, necessarily implies the conservation of energy, namely that *the total energy* $\boldsymbol{E}$ of the solid, composed of *the elastic energy* $\boldsymbol{F}^{él}(\tau)$ of expansion and *the kinetic energy* $\boldsymbol{T}(\tau)$ of expansion, is a constant.

The total kinetic energy $\boldsymbol{T}$ of volume expansion is necessarily linked to the speed of expansion, which can be characterized by the velocity $\vec{\phi}_U(t)$ of the surface of the sphere (figure 3.6). The kinetic energy $\boldsymbol{T}$ can then be obtained by summing (by integration) over the whole sphere the kinetic energy of the lattice sites located in a spherical cap of radius r and thickness dr. The speed of expansion in the spherical cap is simply given by $\phi(r)=\phi_U\ r/R_U$ since the volume expansion was assumed to be homogeneous. From the expression of the kinetic energy $\boldsymbol{T}$ thus obtained as a function of $\vec{\phi}_U$ and of the total number N of sites contained in the sphere, the dependence of $\phi_U(\tau)$ is deduced as a function of $\boldsymbol{T}(\tau)$. On the other hand, like $\boldsymbol{T}(\tau)=\boldsymbol{E}-\boldsymbol{F}^{él}(\tau)$, we finally get *the dependence of* $\phi_U(\tau)$ *as a function of* $\boldsymbol{F}^{él}(\tau)$. But the total elastic energy can be calculated quite easily from the state function $U^{déf}$ of the lattice since τ is homogeneous in the space of the sphere and the other distortion tensors are zero. With the expression of $\boldsymbol{F}^{él}(\tau)$ calculated and reported in figure 3.6, one can directly expressed $\phi_U(\tau)$ as a function of τ and of the total energy $\boldsymbol{E}/N$ per lattice site. The expression shown in figure 3.6 shows that it is possible to plot ϕ_U as a function of τ, for various values of $\boldsymbol{E}/N$. As this expression also depends on the two elastic constants K_0 and K_1, we will get eight different graphs, shown in figure 3.8, depending on whether these two elastic constants are positive, zero or negative, knowing that the graph in the case where the two constants were zero has no sense.

As an example, consider the case of the lattice for which $K_0>0$ and $K_1>0$. The relations of figure 3.6 allow to deduce its *"cosmological behavior"* as represented in figure 3.7:

- if $\boldsymbol{E}<0$, the lattice presents a single possible trajectory, entirely located in the domain $\tau>0$, and which corresponds to expand and contract indefinitely between two extreme values of τ,
- if $0<\boldsymbol{E}<\boldsymbol{F}^{él}_{max}$, the network presents two possible trajectories: the first corresponds to expand and contract indefinitely between a positive value and a negative value of τ, and the second corresponds to oscillate indefinitely between a negative value of τ and an expansion tending towards $\tau\to-\infty$,
- if $\boldsymbol{E}>\boldsymbol{F}^{él}_{max}$, the lattice presents a single very interesting possible trajectory. It pulsates indefinitely from a *big bang* to a *big crunch*. The big bang is followed by a phase of rapid expansion, followed by a deceleration, then again an expansion at increasing speed, and suddenly a reversal of the speed of expansion, to contract again by following in opposite direction every stages traveled during expansion. The contraction ends with a big crunch, which can only be followed by a bigbang since the lattice has then accumulated a total kinetic energy equal to E, phenomenon called *"big bounce"*.

In the case of this lattice, one also notes the existence of domains of volume expansion presenting different behaviors concerning the longitudinal waves: a domain where it coexists transverse and longitudinal waves, for $\tau>\tau_{0cr}=K_0/2K_1-2K_2/3K_1-1$, and a domain where there exists only transverse waves and localized longitudinal eigen vibration modes, for $\tau<\tau_{0cr}=K_0/2K_1-2K_2/3K_1-1$. And the area where there are no longitudinal waves

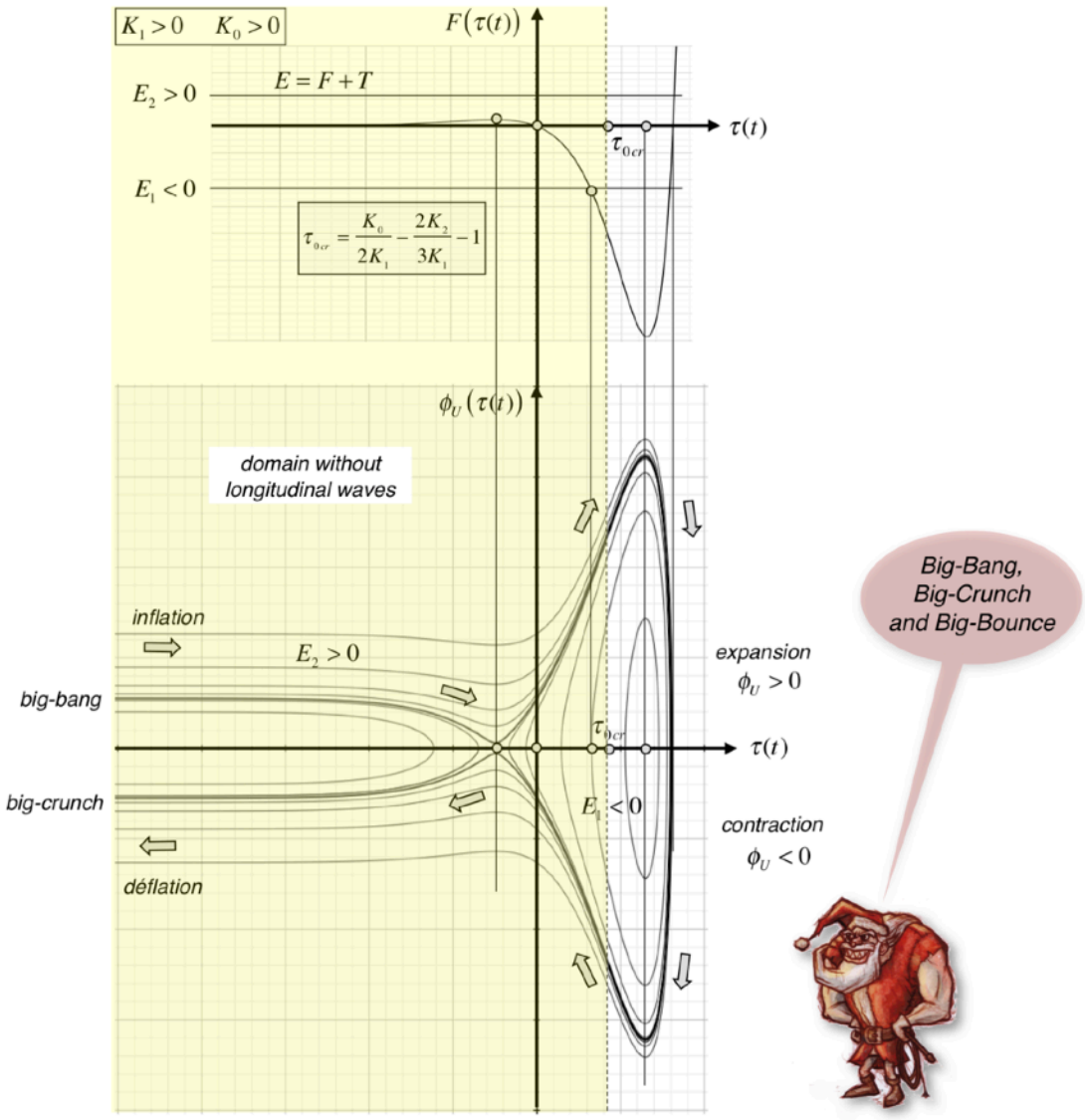

Figure 3.7 *- «cosmological behaviour» of the elastic energy* $\boldsymbol{F}^{\acute{e}l}(\tau)$ *of expansion and of the velocity* $\vec{\phi}_U(\tau)$ *of expansion of a cosmological lattice with* $K_0 > 0$ *and* $K_1 > 0$

corresponds precisely to the area of big bang, inflation, slowing inflation, followed finally with an acceleration of expansion.

In Figure 3.8, we have reported the eight different behaviors that it is possible to obtain with a cosmological lattice, according to the values that the modules K_0 and K_1 can take. Also shown in this figure are the areas of expansion in which the longitudinal waves cannot exist.

We can see that there are *four really different "cosmological behaviors"*, three of which have much more convincing analogies with what we know about the cosmology of the real Universe:
- the cosmological lattices with $K_1 < 0$ which are shown in figures 3.8 *(a), (c)* and *(d)*. These three types of lattices show a big bang followed by high-speed inflation, a slowdown in inflation and finally an expansion at increasing speed towards $\tau \longrightarrow +\infty$, stages which follow one another in perfect order. The disappearance of longitudinal waves occurs in these lattices for expansions greater than a critical value τ_{0cr} which depends on the value of the shear modulus

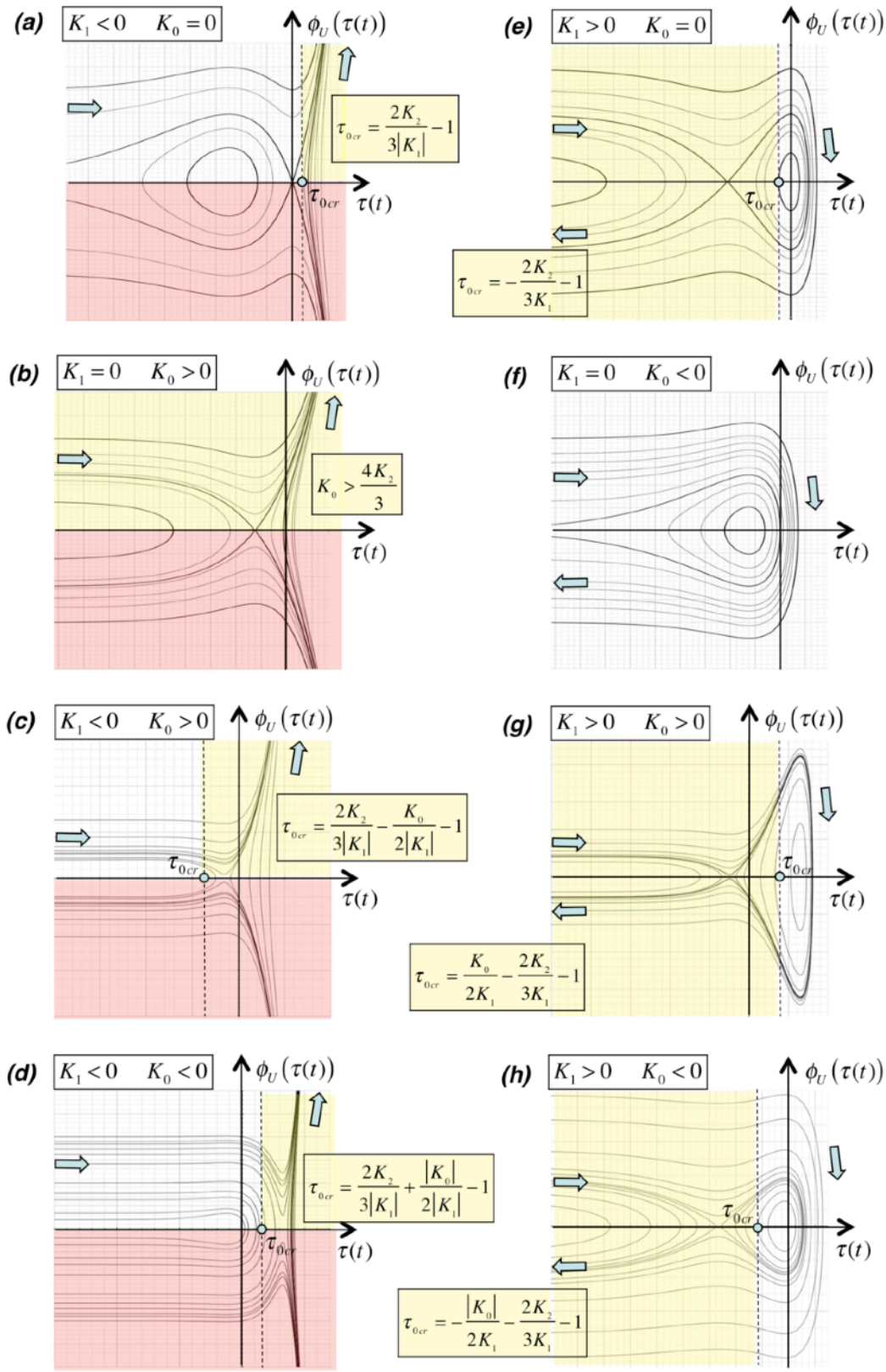

Figure 3.8 *- All the possible cosmological behaviours of a cosmological lattice, according to the values of* K_0 *and* K_1 *: from* ***(a)*** *to* ***(d)*** *the solids with an infinite expansion, and from* ***(e)*** *to* ***(h)*** *the solids oscillating from big-bang to big-crunch*

$K_2 > 0$,

- the cosmological lattice of figure 3.8 *(b)*, with $K_1 = 0$ and $K_0 > 0$, for which there never exists longitudinal waves provided that $K_0 > 4K_2/3$, which makes it a very simple and very interesting case to describe the cosmological behavior of the real Universe,

- the cosmological lattices with $K_1 > 0$ or $K_1 = 0$ and $K_0 < 0$ which are shown in figures 3.8 *(e), (g)* and *(h)*. These three types of lattices go well through the four stages of the cosmology of the real Universe, in the absence of longitudinal waves (a "big-bang" starting from a singularity of space-time, followed by a period of very rapid inflation, then a slowdown in inflation, followed by an expansion whose speed seems to increase well over time), before entering a phase of expansion during which the longitudinal waves appear, and which precedes a symmetrical phase of contraction returning to the state of singularity for $\tau \to -\infty$ («big-crunch"). In this case, there is indeed a region of the diagram, for $\tau < \tau_{0cr}$, which lies in the domain where there are no longitudinal waves, and in which the lattice is expanding at increasing speed. Note that the lattice in figure 3.8 *(g)* could be an excellent candidate to describe the cosmological behavior of the real Universe, because all of its elastic moduli are positive,

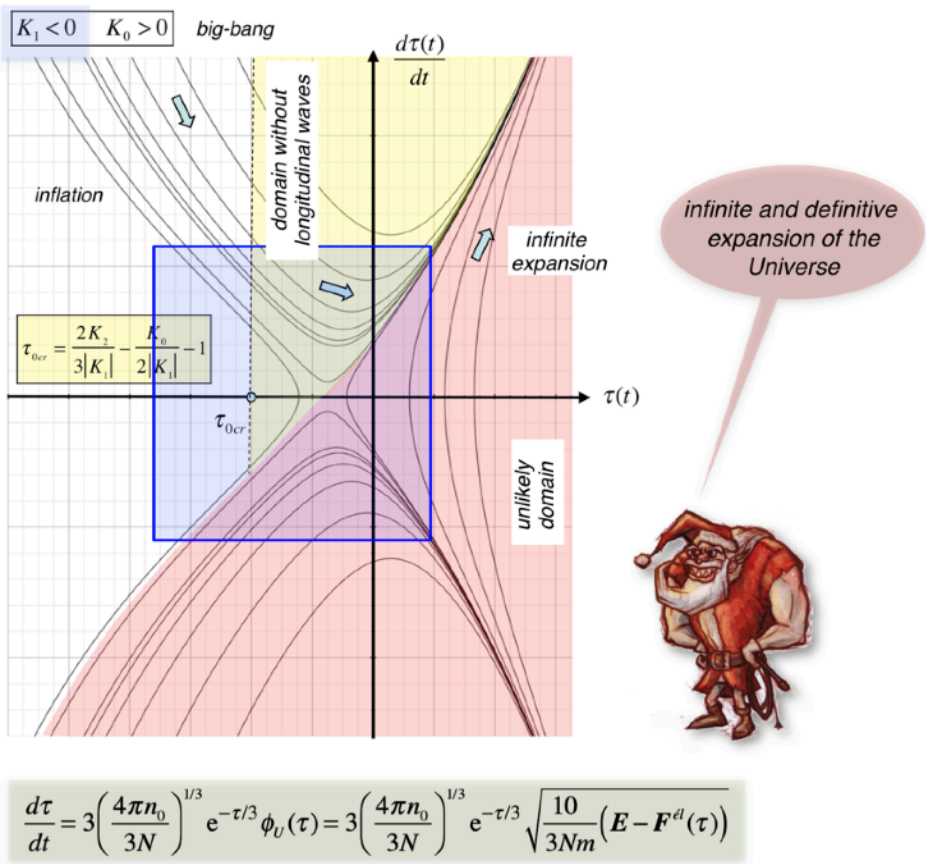

Figure 3.9 *- «cosmological behavior» of the speed of cosmological expansion $d\tau / dt$ as a function of τ of an imaginary cosmological lattice with $K_1 < 0$.*

- finally, the cosmological lattice of figure 3.8 *(f)*, with $K_1 = 0$ and $K_0 < 0$, does not present the stages corresponding to the cosmological evolution of the real Universe, and it always presents longitudinal waves. It is not able to describe the cosmological behavior of the real Universe.

The "cosmological behavior" of a lattice can be illustrated even more clearly by calculating the speed $d\tau / dt$ of the volume expansion as a function of the volume expansion τ, as shown for case *(c)* with τ in figure 3.9, and for case *(g)* with $K_1 > 0$ in figure 3.10. The behavior of the speed $d\tau / dt$ of volume expansion as a function of τ can be deduced from the knowledge of $\boldsymbol{F}^{él}(\tau)$ as shown by the relation reported in figure 3.9.

Figures 3.9 and 3.10 are very interesting, because they clearly show the existence of an initial stage of extremely rapid inflation of volume expansion τ in cosmological lattices since $d\tau / dt \to \pm\infty$ for $\tau \to -\infty$, just after the big-bang stage or just before the big-crunch stage, and that the speed of volume expansion or contraction goes through a minimum before accelerating again, just after the inflation stage or just after the re-contraction stage.

It goes without saying that the modeling used here to describe the "cosmological behaviors" of imaginary lattices is extremely simplified, and one could even qualify it as simplistic. In fact, it is essentially the initial hypothesis of a homogeneous volume expansion throughout the lattice that can be questioned, because with this hypothesis we eluded the two major problems which would lead in principle to much more complicated models: the fact that the solid is subjected to the Newtonian dynamics in the absolute space of the ***GO***, and the fact that one should in

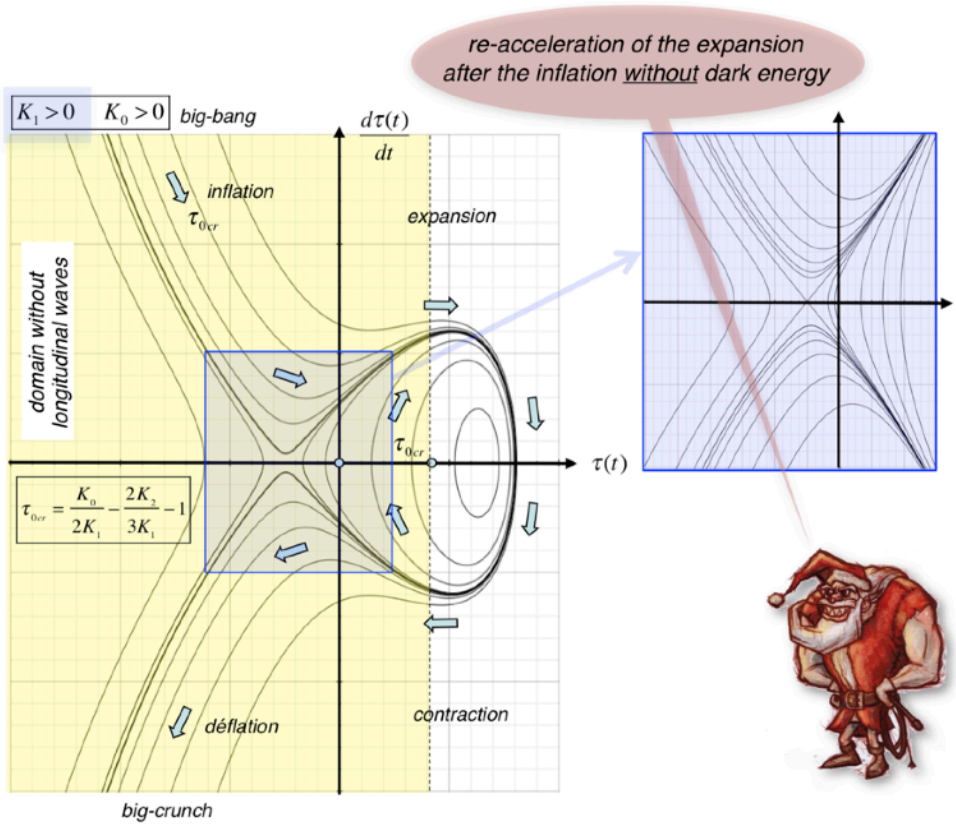

Figure 3.10 *- «cosmological behavior» of the speed of cosmological expansion $d\tau / dt$ as a function of τ of an imaginary cosmological lattice with $K_1 > 0$.*

principle have placed a condition on the nullity of the pressure at the external limit of the solid. But despite the extreme simplifications of our modeling, the global behaviors predicted in figures 3.8 to 3.10 must still remain fairly close to the behaviors that could have been obtained by a more realistic treatment of the problem.

It is obviously not possible to choose here the cosmological lattice which would come closest to the known cosmological evolution of the real Universe. But from a philosophical point of view and from a common sense point of view, the cosmological lattices of figures 3.8 *(e)* to *(h)*, which present a big-bang followed by a big crunch, and therefore ultimately a big- bounce seem much more satisfactory for a Cartesian spirit than the lattices of figures 3.8 *(a)* to *(d)*, which present an infinite and unique expansion. We can therefore make *a "philosophical" conjecture* here, shown in figure 3.11.

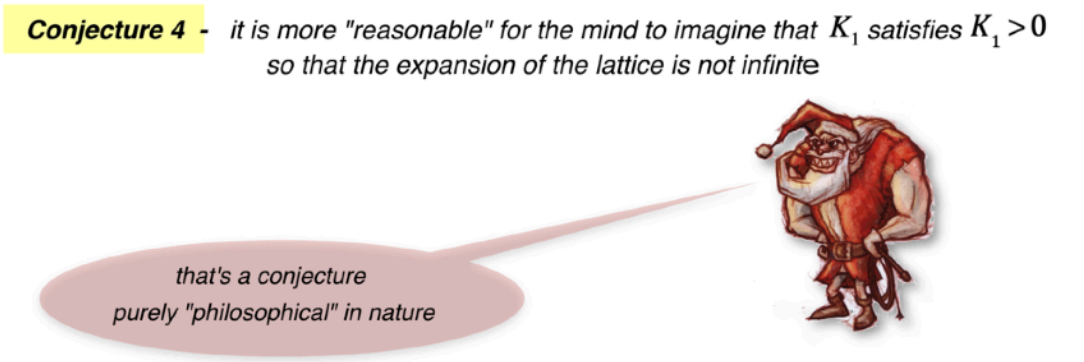

Figure 3.11 *- the fourth common sense conjecture*

As for the value of K_0, nothing currently allows us to propose a positive, zero or negative value, because the cases illustrated in Figures 3.8 *(e), (g)* and *(h)* are all three very interesting.

Note also that the elastic energy $\boldsymbol{F}^{él}(\tau)$ contained in the cosmological lattice could well correspond to the famous *"dark energy"* of the astrophysicists, introduced to explain the increase in the speed of the expansion of the Universe recently observed experimentally, since it is this elastic energy which is entirely responsible for the increase in the speed of the volume expansion via the relation reported in figure 3.9.

It is again remarkable to note that these behaviors of the volume expansion of a sphere of cosmological lattice are exclusively due to *the development of the internal energy per unit of volume* which we used for this one. Indeed, if we try to look at what would happen in the case of a perfect solid for which the internal energy would be written as a development expressed by lattice site, we would see that the behavior of expansion is radically different like shown in figure 3.12:

- *if the modulus* k_1 *of the imaginary perfect solid is positive*, this can only oscillate indefinitely between a state of minimum volume expansion $\tau_{\min}$ and a state of maximum volume expansion $\tau_{\max}$, as illustrated by the first figure 3.12. If one reports in the diagrams $\boldsymbol{F}^{él}(\tau)$ and $\vec{\phi}_U(\tau)$, the critical value $\tau_{0cr} = 1 + 2k_2 / 3k_1 > 1$ of τ_0 below which there are longitudinal waves in this perfect lattice, one notes that during its *"cosmological evolution"* the solid will pass

alternately from a domain ($\tau \leq \tau_{0cr}$) where there coexist transverse and longitudinal waves, with another domain ($\tau \geq \tau_{0cr}$) where there exist only transverse waves and localized modes of longitudinal vibrations. But in this domain without longitudinal waves, the speed of expansion can only slow down, which is in disagreement with the observations made on the current Universe.

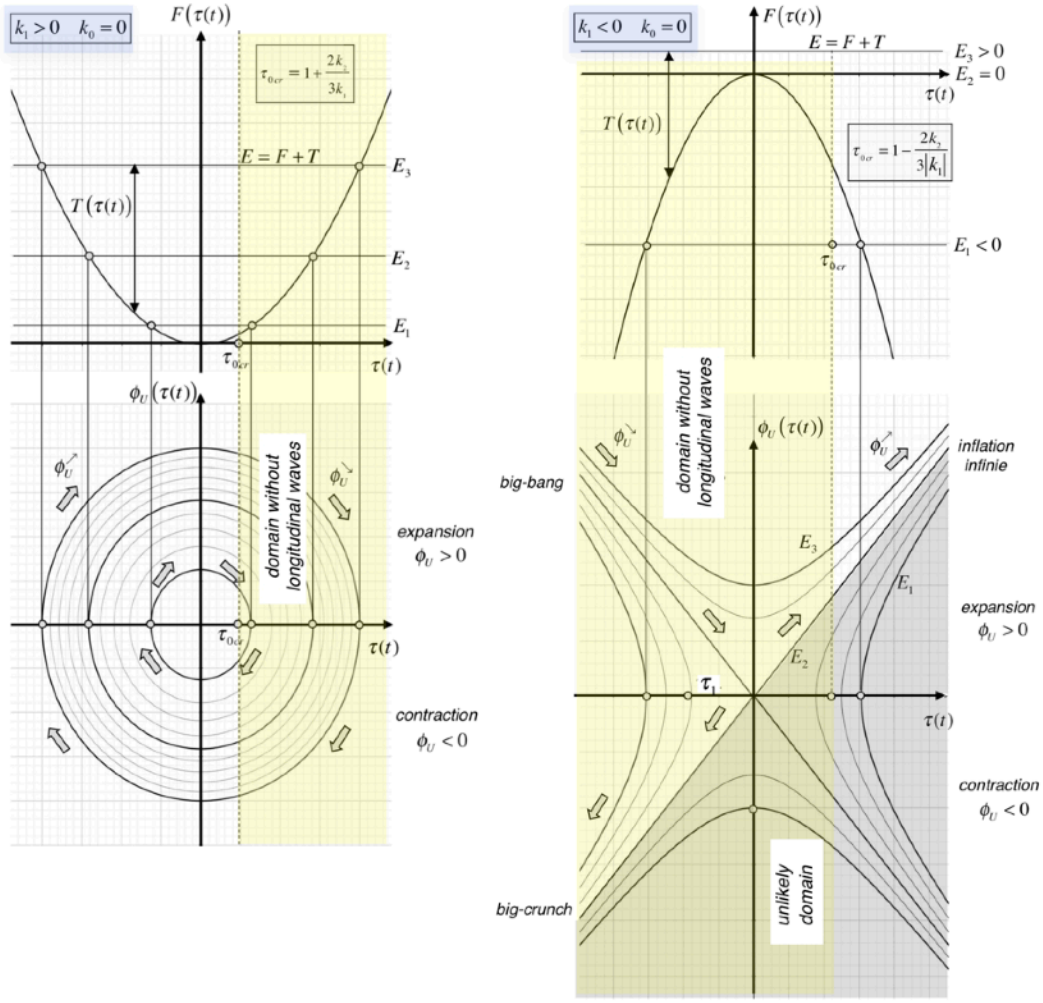

Figure 3.12 *- «cosmological behaviors» of the elastic energy* $\boldsymbol{F}^{él}(\tau)$ *of expansion and of the speed* $\vec{\phi}_U(\tau)$ *of expansion of an imaginary perfect solid whose internal energy would be written as a developement expressed by lattice site, for* $k_1 < 0$ *and* $k_1 > 0$

- *if the modulus* k_1 *of the imaginary perfect solid is negative*, depending on the value of the total energy $\boldsymbol{E}$, this solid could have several different *"cosmological behaviors"* as well illustrated by the second figure 3.12. If $\boldsymbol{E} \leq 0$, it can contract and expand in a back and forth movement between $\tau \to -\infty$ and τ_1 , or then expand indefinitely from $\tau = 0$. Note that it is difficult to imagine a solid that would evolve by contracting since $\tau = \infty$, which is why these behaviors are reported in a gray area. If $\boldsymbol{E} \geq 0$, it can expand indefinitely from $\tau \to -\infty$. In this

case, the longitudinal waves appear in the domain $\tau \geq \tau_{0cr} = 1 - 2k_2 / 3|k_1| < 1$.

The different "cosmological behaviors" deduced for an imaginary perfect solid can also be compared with the cosmological behavior which is currently attributed to our real Universe. Indeed, in the case of the real universe, we have a system which does not present longitudinal waves, as shown by the theory of general relativity, and which pursues, in the light of the last observations, a cosmological evolution in several stages: a “big-bang” based on a singularity of space-time, followed by a period of very rapid inflation, then a slowdown in inflation, followed, according to very recent experimental observations, by an expansion whose speed seems to increase over time. This last stage is the one that would correspond to the current state of our Universe.

Among the *"cosmological behaviors"* deduced from the *perfect solid*, only the perfect solid with $k_1 < 0$ presents any analogy with the cosmological behavior of the real Universe. Indeed, the perfect solid with $k_1 < 0$, in the case where E is greater than zero (second figure 3.12), traverses well the stages of big bang, inflation, slowing of inflation and expansion at increasing speed in the area where there are no longitudinal waves. But for this solid, the stage of expansion at increasing speed inevitably continues towards $\tau \rightarrow +\infty$, which does not satisfy our fourth conjecture dictated by common sense.

These results in the case of the perfect solid whose internal energy would be written as a development expressed by lattice site are far from satisfactory if we compare them with the known expansion of the universe. This therefore justifies *again* the conjecture 1 that we had posed in figure 3.1, since it is this one which allows the existence of the cosmological behaviors reported in figures 3.7 to 3.10, and especially which allows the existence of the curvature of the wave rays by the volume expansion gradients of figures 3.4 and 3.5.