Chapter 11

Quantum Physics

In Figure 7.0, we have seen that when the mass density of a singularity or cluster of singularities exceeds a certain critical value, there are no static solutions for the field of internal expansion disturbances, *which means that these internal disturbances must necessarily become dynamic disturbances*. Intuitively, it is intuitively suspected that the existence of such dynamic solutions of Newton's second partial equation (figure 5.1) may well be related to the appearance of quantum physics, i.e. the existence of localized temporal fluctuations in the expansion field of the perfect cosmological lattice when it does not exhibit longitudinal wave propagation in the domain $\tau_0 < \tau_{0cr}$. We will show in this chapter that a wave function, deduced directly from Newton's second partial equation of the lattice expansion perturbations, is indeed intimately related to the mobile topological singularities of the lattice, whether these are clusters of elementary loops or isolated elementary loops.

We will thus be able to give a quite classical and rather simple *"wave" interpretation of quantum physics*: *the quantum wave function would in fact represent the amplitude and the phase of the gravitational fluctuations coupled to the topological singularities*. This interpretation then implies that the square of the amplitude of the normalized wave function is indeed related to the probability of the presence of the associated topological singularity.
In the wake of this, we also find *Heisenberg's uncertainty principle, the notions of bosons, fermions and indistinguishability, Pauli's exclusion principle*, and the path towards a physical understanding of intriguing phenomena such as *quantum entanglement and quantum decoherence*.

Thus, from figure 7.0, we deduce that the critical value of the mass density at which expansion perturbations become dynamic is nothing more than *the quantum decoherence limit of this singularity or cluster of singularities*.

Relativistic wave equation of gravitational expansion field fluctuations beyond the quantum decoherence limit

Above *the quantum decoherence limit*, i.e. for values of the mass density of a singularity or cluster of singularities greater than the critical mass density (figure 7.0), suppose the existence of dynamic longitudinal fluctuations in the cosmological lattice. These fluctuations must obviously satisfy the dynamic version of Newton's second partial equation in figure 5.1. We obtain at first order in $\tau^{(p)}$, by also taking into account the geometro-kinetic equation for $\tau^{(p)}$, and by emitting the hypotheses that $\tau_0 << \tau_{0cr}$, $K_2 << K_0$ and $2K_1(1+\tau_0) << K_0$, the equation managing the gravitational fluctuations, represented in figure 11.1.

This equation is perfectly realistic, in that it is derived from Newton's equation of the lattice. However, it cannot be solved in this form because it would require to know the function

$F_{dist}^{ch}(\vec{r},t)+F_{pot}^{ch}(\vec{r},t)$ associated with the topological singularity, which depends in particular on the trajectory of the singularity within the network, a trajectory that we do not know a priori. We can therefore say that this function $F_{dist}^{ch}(\vec{r},t)+F_{pot}^{ch}(\vec{r},t)$ is in fact *a hidden variable of the problem*. It will thus be necessary, as we have already done in chapter 7 in the static case by introducing a mean value $\overline{\tau}_{int}^{(E)}$ of the static internal field, to find here again *a subterfuge* to find some form of solution to this dynamic equation.

Let us first propose a simple solution, localized and independent of the time of this equation for the wave function $\psi_{localisée}\left(\vec{r}\right)$, assuming that it is a simple exponential decay with spans $\delta_1,\delta_2,\delta_3$ in the three directions of space, and that the pulsation ω_f is a real constant independent of time and space (figure 11.1). By introducing this solution into the wave equation that we have just established, we obtain the relation which must exist between the pulsation ω_f of the fluctuations and the ranges $\delta_1,\delta_2,\delta_3$ of these fluctuations. There then appears a localized gravitational fluctuation $\underline{\tau}_{localisée}^{(p)}$ which corresponds to a localized, non-damped and pulsating regime with a pulsation ω_f which decreases symmetrically and exponentially in the vicinity of the origin, with spatial ranges equal to δ_1,δ_2 and δ_3 along the 3 axes, which are correlated with each other and which decrease with the pulse frequency.

We therefore deduce that, in a perfect cosmological lattice satisfying the hypotheses that we have put forward, there can perfectly exist stable and localized fluctuations of vibrations of the volume expansion. We will return to this subject in more detail in chapter 14.

Let us imagine that mobile topological singularities within the lattice, such as clusters of elementary loops or isolated elementary loops as described in the previous chapters, are also associated with dynamic longitudinal gravitational fluctuations, which should obviously satisfy the dynamic version of Newton's second partial equation in the vicinity of the topological singularity.

It is known that, when the energy density of a singularity is lower than the critical value of the quantum decoherence limit, the second Newton partial equation reveals static gravitational perturbations of the expansion field within stationary clusters, perturbations that are directly dependent on the elastic distortion energy of these singularities. And in chapter 7, it was shown that this internal expansion disturbance field has a mean value $\overline{\tau}_{int}^{(E)}$ which is directly responsible for a static external gravitational field of expansion disturbances, in the form of $\tau_{ext\ LD}^{cluster}(r)\cong -4\boldsymbol{G}_{grav}\boldsymbol{M}_0^{cluster}/c_t^2 r$, which also depends on the elastic distortion energy of the singularity or cluster of stationary singularities via the parameter $4\boldsymbol{G}_{grav}/c_t^2$.

On the other hand, when the energy density of a singularity is *higher than the critical value of the quantum decoherence limit*, the gravitational disturbances can only be dynamic, and must then satisfy the Newtonian relationship shown in Figure 11.1, which also depends on the elastic and potential energy densities of the singularity.

There are therefore strong presumptions that, beyond the quantum decoherence limit, the field of dynamic gravitational perturbations $\tau^{(p)}$ of the expansion outside a moving singularity can be represented by *a subterfuge of imagining a similar pattern to that given for the isolated gravitational fluctuations in figure 11.1, but which this time also depends on the elastic distortion energy of the singularity, but probably also on the energy associated with the motion of the singularity*. Now we know that a moving topological singularity in the lattice is entirely characterized by a *total relativistic energy* $E_{\boldsymbol{v}}$ and a *total relativistic momentum* $\vec{\boldsymbol{P}}_{\boldsymbol{v}}$.

Therefore, the pulsation $\underline{\omega}_f$ of the associated expansion fluctuations should certainly also depend on the relativistic energy $E_v(\vec{r},t)$ of the singularity and its relativistic momentum $\vec{P}_v(\vec{r},t)$. Let us therefore imagine that the topological singularities moving within the lattice, such as clusters of elementary loops or isolated elementary loops as described in the previous chapters, are also associated with dynamic longitudinal gravitational fluctuations, which should obviously satisfy the dynamic version of Newton's second partial equation outside the topological singularity.

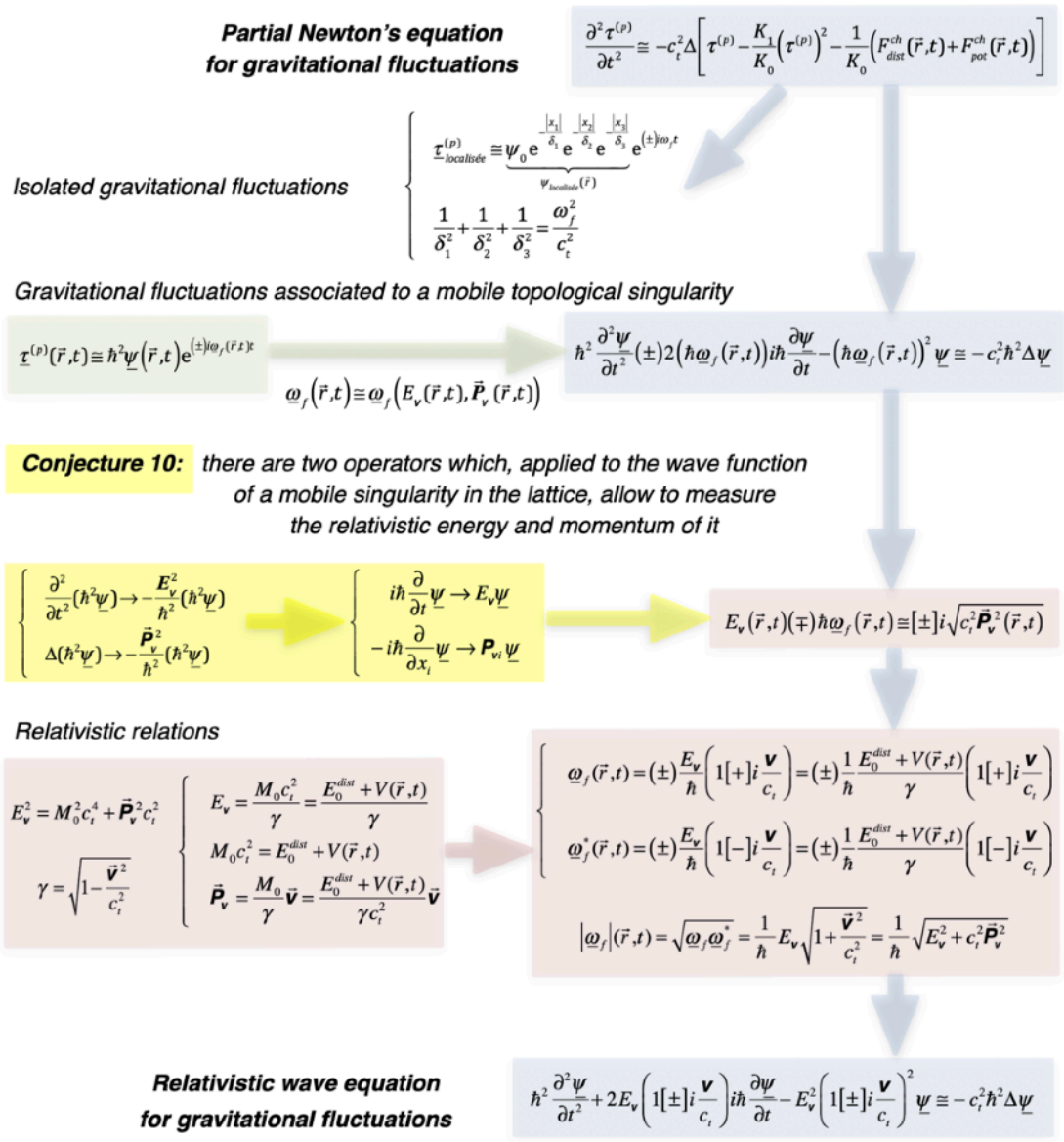

Figure 11.1 *- From Newton's partial equation for gravitational fluctuations to the relativistic wave equation for the fluctuations associated with a moving singularity*

We can then propose a solution of gravitational perturbations of complex pulsation $\underline{\omega}_f$, which we will write for convenience in complex formulation in the form $\underline{\tau}^{(p)} \cong \hbar^2 \underline{\psi}\, e^{(\pm)i\underline{\omega}_f t}$, where $\hbar^2$ is an arbitrary constant for the moment, $e^{(\pm)i\underline{\omega}_f t}$ corresponds to *the oscillation of the*

fluctuation and *the complex wave function* $\underline{\psi}(\vec{r},t)$ *contains the phase and amplitude information for this oscillation*. By introducing this solution into Newton's equation for lattice fluctuations, we obtain a wave equation for the complex function $\underline{\psi}(\vec{r},t)$ in the form shown in figure 11.1.

If the frequency $\underline{\omega}_f(\vec{r},t)$ of the gravitational fluctuations effectively depends on $E_{\boldsymbol{v}}(\vec{r},t)$ and $\vec{\boldsymbol{P}}_{\boldsymbol{v}}(\vec{r},t)$, these two quantities must also be implicitly contained in the temporal and spatial behaviors of the complex wave function $\underline{\psi}(\vec{r},t)$ associated with the mobile singularity. However, this wave function $\underline{\psi}(\vec{r},t)$ associated with the wave equation in figure 11.1 actually recalls the wave function appearing in quantum physics, and quantum physics says that it is possible *to define operators* allowing to measure $E_{\boldsymbol{v}}$ and $\vec{\boldsymbol{P}}_{\boldsymbol{v}}$ from the quantum wave function $\underline{\psi}(\vec{r},t)$.

By analogy with quantum physics, we will therefore *conjecture a priori operators with similar properties:* this will be conjecture 10 of our approach, reported in figure 11.1. In fact, we are going to hypothesize that *the second derivative with respect to the time of the envelope* $\hbar^2\underline{\psi}$ *of the oscillations of* $\underline{\tau}^{(p)}$ *is directly correlated with the square of the total relativistic energy* $E_{\boldsymbol{v}}$ of the singularity, and that *the Laplacian (the second derivative with respect to space) of the envelope* $\hbar^2\underline{\psi}$ *of the oscillations of* $\underline{\tau}^{(p)}$ *is directly correlated with the square of the total relativistic momentum* $\vec{\boldsymbol{P}}_{\boldsymbol{v}}$. In the relations associated with this conjecture, we introduced, as in quantum physics, a constant $\hbar$ allowing to normalize the partial derivatives of the wave function to energetic terms (and which is, in the case of quantum physics, nothing other than the Planck's constant). It is quite simple to verify that these operators, applied once to the wave function, provide the usual operators of quantum physics, as illustrated in figure 11.1.


Max Planck
(1858-1947)

Conjecture 10 that we have assumed here has a physical meaning that can be explained very simply. Indeed, suppose that the envelope $\hbar^2\underline{\psi}$ of the oscillations of $\underline{\tau}^{(p)}$ is of oscillatory type, of the form $\hbar^2\underline{\psi} = \hbar^2\psi_0\sin(\Omega t - kx)$. In this case, conjecture 10 simply implies that the pulsation $\Omega = 2\pi f = \boldsymbol{E}_{\boldsymbol{v}}/\hbar$ of the envelope is proportional to *the total relativistic energy* $E_{\boldsymbol{v}}$ of the singularity and that the wave number $k = 2\pi/\lambda = \left|\vec{\boldsymbol{P}}_{\boldsymbol{v}}\right|/\hbar$ of the envelope is proportional to *the relativistic momentum* $\left|\vec{\boldsymbol{P}}_{\boldsymbol{v}}\right|$ of the singularity, with a proportionality factor equal to the inverse of the Planck constant.

Thanks to the arbitrary constant $\hbar^2$ introduced into the solution for $\underline{\tau}^{(p)}(\vec{r},t)$, the wave equation for $\underline{\psi}(\vec{r},t)$ is then directly composed of terms corresponding to energy squares. We deduce that each term of this equation must represent the product of the square of an energy by the dimensionless wave function $\underline{\psi}(\vec{r},t)$. We deduce in particular that *the term* $\hbar\omega_f$ *has the dimension of an energy.*

On the other hand, the second derivative operator of time provides the square of *the total relativistic energy* $E_{\boldsymbol{v}}$ that the singularity would have at the place where the operator is applied, and the Laplacian operator provides the square of *the total relativist momentum* $\vec{\boldsymbol{P}}_{\boldsymbol{v}}$ that the singularity would have at the place where the operator is applied. From this definition of the second derivative and Laplacian operators, we deduce the operators directly supplying the total

energy of the singularity and the components of the momentum of the singularity at a given location.

With these "a priori" conjectured operators based on quantum physics, we can try to apply them to the wave relation deduced from Newton's second partial equation. We thus obtain the relation which should exist between *the relativistic energy* $E_{\boldsymbol{v}}$ of the singularity, *the energy of movement* $\sqrt{c_t^2 \vec{\boldsymbol{P}}_{\boldsymbol{v}}^2}$ of the singularity and *the pulsation* $\underline{\omega}_f(\vec{r},t)$ *of the gravitational fluctuations associated with the singularity*, which now *really becomes a complex number, in the form* $E_{\boldsymbol{v}}(\vec{r},t)(\mp)\hbar\underline{\omega}_f(\vec{r},t) \cong [\pm] i\sqrt{c_t^2 \vec{\boldsymbol{P}}_{\boldsymbol{v}}^2(\vec{r},t)}$. It should be noted here that the double signs $(\mp)$ and $[\pm]$ appearing in this relation are independent of each other because the first $(\mp)$ comes from the expression $\underline{\tau}^{(p)} \cong \hbar^2 \underline{\psi}\, \mathrm{e}^{(\pm) i\underline{\omega}_f t}$ and the second is introduced by taking the square root of $-c_t^2 \vec{\boldsymbol{P}}_{\boldsymbol{v}}^2(\vec{r},t)$.

In the case of a singularity which moves *at relativistic speed*, this one must satisfy the principal relativistic relations obtained in chapter 6 and reported in figure 11.1, in particular the important relation $E_{\boldsymbol{v}}^2 = M_0^2 c_t^4 + \vec{\boldsymbol{P}}_{\boldsymbol{v}}^2 c_t^2$ which makes the link between *the total relativistic energy* $E_{\boldsymbol{v}}$, *the relativistic momentum* $\vec{\boldsymbol{P}}_{\boldsymbol{v}}$ and *the mass at rest* M_0 of the singularity. These relativistic expressions lead to the relation giving *the complex pulsation* $\underline{\omega}_f(\vec{r},t)$ of the gravitational fluctuations associated with a topological singularity, either an elementary loop, or a cluster of elementary loops linked together, moving at speed $\boldsymbol{v}$. The complex relativistic pulsation $\underline{\omega}_f(\vec{r},t)$ therefore presents two conjugate solutions shown in figure 11.1. It is interesting to note that there is indeed an expression justifying the hypothesis that we had put forward, namely that $\underline{\omega}_f(\vec{r},t)$ is a direct function of $E_{\boldsymbol{v}}$ *and* $\vec{\boldsymbol{P}}_{\boldsymbol{v}}$. Indeed, by performing the product of the two combined values of the pulsation, we see that it is the norm $\left|\underline{\omega}_f\right|(\vec{r},t) = (\underline{\omega}_f \underline{\omega}_f^*)^{1/2}$ of the complex pulsation $\underline{\omega}_f(\vec{r},t)$ which is a simple and direct function of the quantities $E_{\boldsymbol{v}}$ and $\vec{\boldsymbol{P}}_{\boldsymbol{v}}$ of the singularity.

The wave equation for $\underline{\psi}(\vec{r},t)$ then has *two relativistic versions* shown in figure 11.1, because of the double sign $[\pm]$ that appears in $(1[\pm] i\boldsymbol{v}/c_t)$. On the other hand, in this expression of the wave equation, the double sign $(\mp)$ no longer appears.

Gravitational perturbations of a massive singularity at relativistic speed

Let us take again the relativistic wave equation of figure 11.1 and try to find a solution for a rather massive singularity which would move *almost freely at relativistic speed* $\boldsymbol{v}$ *more or less constant* in the direction $\boldsymbol{O}x_2$, which implies that $E_{\boldsymbol{v}}(\vec{r},t)$ *varies relatively slowly in time and space*. Under this hypothesis that the total relativistic energy of the singularity varies only slowly in time and space, we can admit that the function $\hbar^2 \underline{\psi}(\vec{r},t)$, which represents the amplitude and the phase of the oscillation with a pulsation $\omega_f(\vec{r},t)$ is in fact a function of the position x_2 along the axis $\boldsymbol{O}x_2$. Let us therefore pose a simple wave solution of the relativistic wave equation along the axis $\boldsymbol{O}x_2$, which does not explicitly depend on time, on the type $\underline{\psi}(\vec{r},t) \cong \psi_0\, \mathrm{e}^{i\underline{k}(\vec{r},t)x_2}$, where the complex wave number $\underline{k}(\vec{r},t)$ also varies very slowly over time and the space. By injecting this solution into *the relativistic wave equation*, we obtain the expression of figure 11.2 for the value of the complex wave number $\underline{k}$ and we note that $\underline{k}(\vec{r},t)$ in fact depends on $\vec{r}$ and t only by the dependence of the relativistic energy $E_{\boldsymbol{v}}(\vec{r},t)$ in $\vec{r}$ and t, which allows to express the wave function $\underline{\psi}(\vec{r},t)$ as well as the expansion fluctuations

associated with the relativistic singularity. By introducing the value of $\omega_f(\vec{r},t)$ obtained in figure 11.1, we get two solutions which really have a physical meaning among four possible solutions, so that, finally, the solution for $\underline{\tau}^{(p)}(\vec{r},t)$ can be written in the form represented in figure 11.2. We can therefore explain the real expansion fluctuations of a relativistic singularity, by taking the real part $\tau_{réel}^{(p)}(\vec{r},t)$ of this expression.

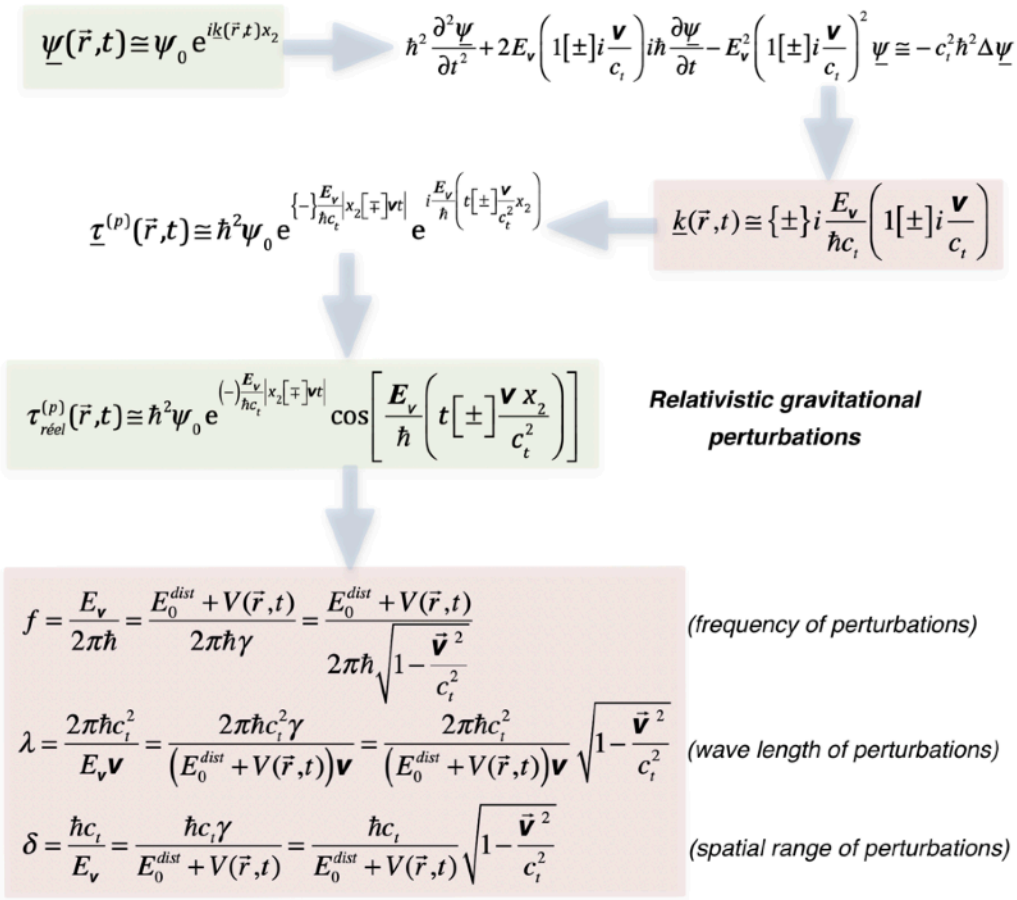

Figure 11.2 *- Associated gravitational perturbations to a massive singularity moving at relativistic speed*

We see that this function $\tau_{réel}^{(p)}(\vec{r},t)$ represents the product of oscillations in time and oscillations in space. Oscillations in time present a frequency f and oscillations in space a wavelength λ. We note that the frequency f of temporal oscillations is an increasing function of the speed $\boldsymbol{v}$ and of the relativistic energy E_v of the singularity, and that it tends towards an infinite value for $\boldsymbol{v} \to c_t$. It depends indirectly on the position $\vec{r}$ of the singularity and on time t via the dependence of the potential $V(\vec{r},t)$ on these quantities. As for the wavelength λ of the spatial oscillations, it decreases as a function of the speed $\boldsymbol{v}$ and of the relativistic energy E_v of the singularity, and it tends towards 0 for $\boldsymbol{v} \to c_t$. It is also modulated in space and time via the potential $V(\vec{r},t)$.

The amplitude of these temporal and spatial oscillations is modulated by an exponentially decreasing envelope on both sides of the average position $x_2(t) = [\pm]\boldsymbol{v}\,t$ of the singularity

(which thus moves in the direction of the axis $\boldsymbol{O}x_2$ or in opposite direction according to the sign + or -). The speed of decrease of the envelope $e^{(-)|x_2[\mp]\boldsymbol{v}t|/\delta}$ is related to *a range* δ *of the envelope of the oscillations*, which decreases when the speed $\boldsymbol{v}$ of the singularity increases and when its relativistic energy $E_{\boldsymbol{v}}$ increases, and it tends towards 0 for $\boldsymbol{v} \to c_t$. It, too, is modulated in space and time via the potential $V(\vec{r},t)$ that the singularity undergoes. All this implies that the gravitational fluctuations associated with the total relativistic energy of the singularity are very short spans and certainly become negligible for massive singularities, such as clusters of linked elementary loops. For example, for an electron at non-relativistic speed, the range of gravitational perturbations associated with its rest energy E_0^{dist} is already tiny, of the order of $2 \cdot 10^{-12}\ m$.

Finally, we note that the dynamic expansion fluctuations associated with the relativistic singularity are contracted along the axis $\boldsymbol{O}x_2$ of the movement of the singularity, as a function of the speed $\boldsymbol{v}$ of the singularity, as we can see on the wavelength λ of the spatial oscillations and on the range δ of the envelope of the oscillations. *These effects strictly correspond to the relativistic effect of contraction of the rulers of a mobile cluster of singularities* described in chapter 6.

Schrödinger's equation of gravitational perturbations of a massive singularity moving at non-relativistic speed in a variable potential

The treatment of the previous section applies to gravitational perturbations associated with a massive singularity which moves at relativistic speeds and which is not subject to a sufficiently strong potential to influence strongly its trajectory. But for a microscopic singularity (a loop of disclination or dislocation for example) in a non-relativistic regime $|\boldsymbol{v}| << c_t$, and subject to a potential $V(\vec{r},t)$ large enough to influence strongly its trajectory, one can wonder what becomes its wave equation. To do this, let's rewrite the relativistic wave equation obtained in figure 11.1 by removing the very small terms in $\boldsymbol{v}/c_t$. This is the first step shown in figure 11.3.


Erwin Schrödinger
(1887-1961)

Then using conjecture 10, the wave equation can transform into a *"reduced form"* containing only the first derivative of time. But the total energy $E_{\boldsymbol{v}}(\vec{r},t)$ of the non-relativistic singularity can be expressed as $E_{\boldsymbol{v}} = M_0 c_t^2 / \gamma \cong M_0 c_t^2$ or as $E_{\boldsymbol{v}} = \left(E_0^{dist} + V(\vec{r},t)\right)/\gamma \cong E_0^{dist} + V(\vec{r},t)$, which allows to rewrite approximately the wave equation in a third form, which we will call *the non-relativistic wave equation*. This wave equation then admits two distinct solutions for gravitational perturbations $\underline{\tau}^{(p)}(\vec{r},t)$ which are distinguished by the sign of the rotating vector.

In this form, the non-relativistic wave equation is very similar to Schrödinger's equation of quantum physics, and it can be shown that it is possible to find this equation. Indeed, let's make a change in wave equation by introducing a wave function $\underline{\psi}_H\left(\vec{r},t\right)$ such that it gives the initial wave function $\underline{\psi}\left(\vec{r},t\right)$ when multiplied by the factor $\exp(-iE_0^{dist}t/\hbar)$. We then obtain a non-relativistic wave equation which is very well known, since it is *the famous Schrödinger's equation of quantum physics*.

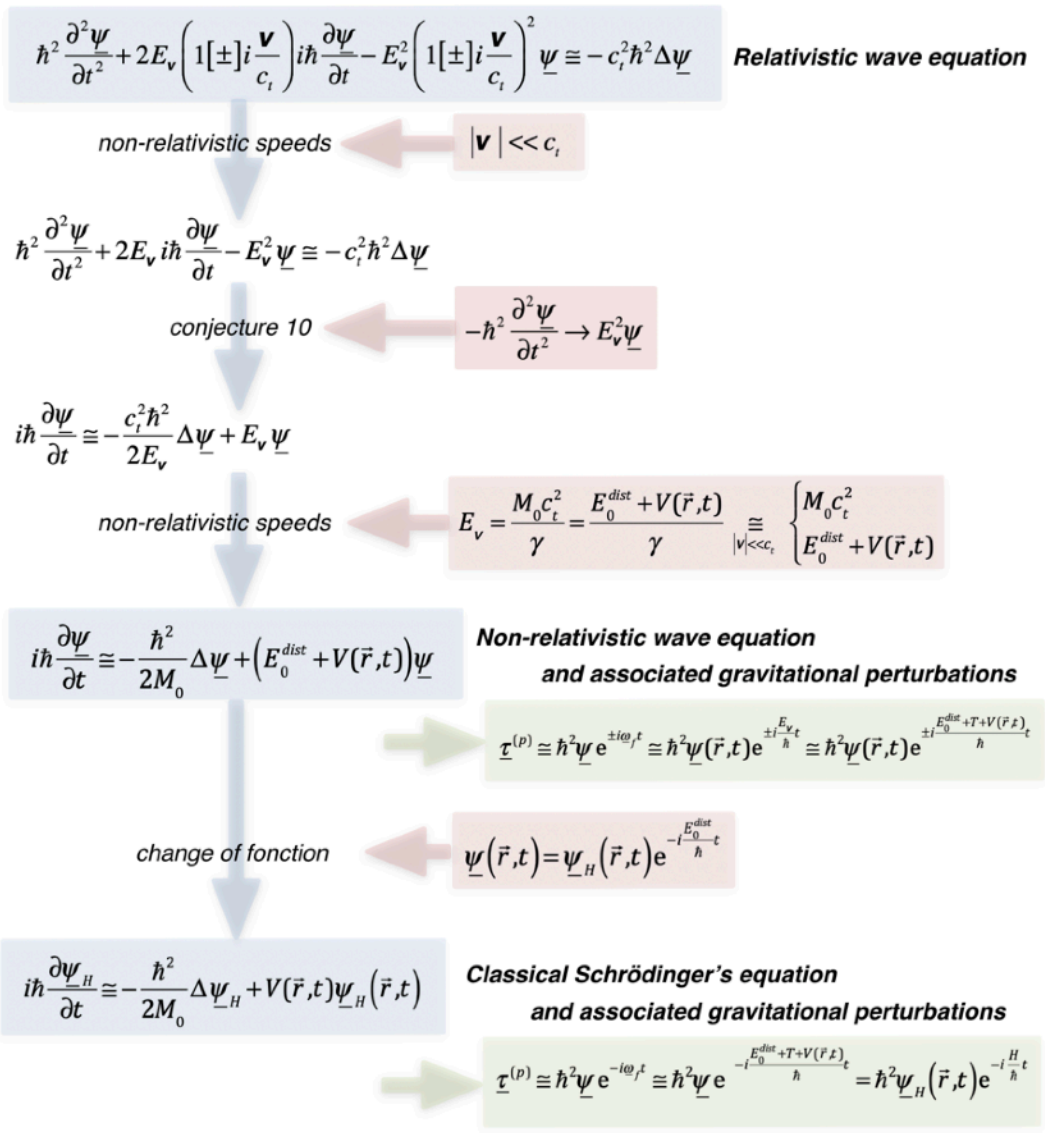

Figure 11.3 *- Non-relativistic wave equation and Schrödinger's equation in the case of gravitational perturbations associated to a massive singularity moving at non-relativistic speed*

By solving this wave equation for a singularity of mass M_0 subject to a potential $V(\vec{r},t)$, we deduce the wave function $\underline{\psi}_H(\vec{r},t)$ which makes it possible to directly find the gravitational perturbations $\underline{\tau}^{(p)}(\vec{r},t)$ associated with this singularity thanks to the relation reported in figure 11.3. But unlike the non-relativistic wave equation that precedes it, this equation, due to the change of the wave function $\underline{\psi}(\vec{r},t)$ by $\underline{\psi}_H(\vec{r},t)$, gives only one solution for the gravitational perturbations $\underline{\tau}^{(p)}(\vec{r},t)$.

The wave function $\underline{\psi}_H(\vec{r},t)$ provided by the Schrödinger's equation is directly related to *the Hamiltonian of the singularity*, namely to the sum of its kinetic energy and its potential energy, since by using the operators defined by conjecture 10 in Schrödinger's equation, we get directly the relation $E_{\underline{\psi}_H} \cong \hat{\boldsymbol{P}}_{\boldsymbol{v}}^2 / 2M_0 + V(\vec{r},t) = T + V(\vec{r},t) = H$.

The wave equation that we have obtained for $\underline{\psi}_H(\vec{r},t)$ corresponds in all points, very exactly, to the time-dependent Schrödinger's equation of quantum physics *for a non-relativistic particle*, provided that *the universal constant* $\hbar$ *that we have conjectured is effectively the Planck constant of quantum physics*. This perfect similarity is obviously not fortuitous and allows *for the first time* to give a very understandable interpretation of quantum physics, by saying that «the Schrödinger's equation is a wave equation deduced from the second partial Newton equation of a perfect cosmological lattice, in the domain $\tau_0 << \tau_{0cr}$, which makes it possible to calculate the envelope $\underline{\psi}_H(\vec{r},t)$ of the dynamic gravitational fluctuations $\underline{\tau}^{(p)}(\vec{r},t)$ associated with a topological singularity of mass M_0 and subjected to a potential $V(\vec{r},t)$ ».

The standing wave equation of a singularity placed in a static potential

If the potential in which the topological singularity is placed is a static potential $V(\vec{r})$, the left term of the non-relativistic wave equation reported in figure 11.4 is an operator giving the total energy E_v of the singularity, which must obviously be a constant since the singularity moves into a static potential. Using conjecture 10 to make the time derivative disappear, we obtain a time-independent wave equation.

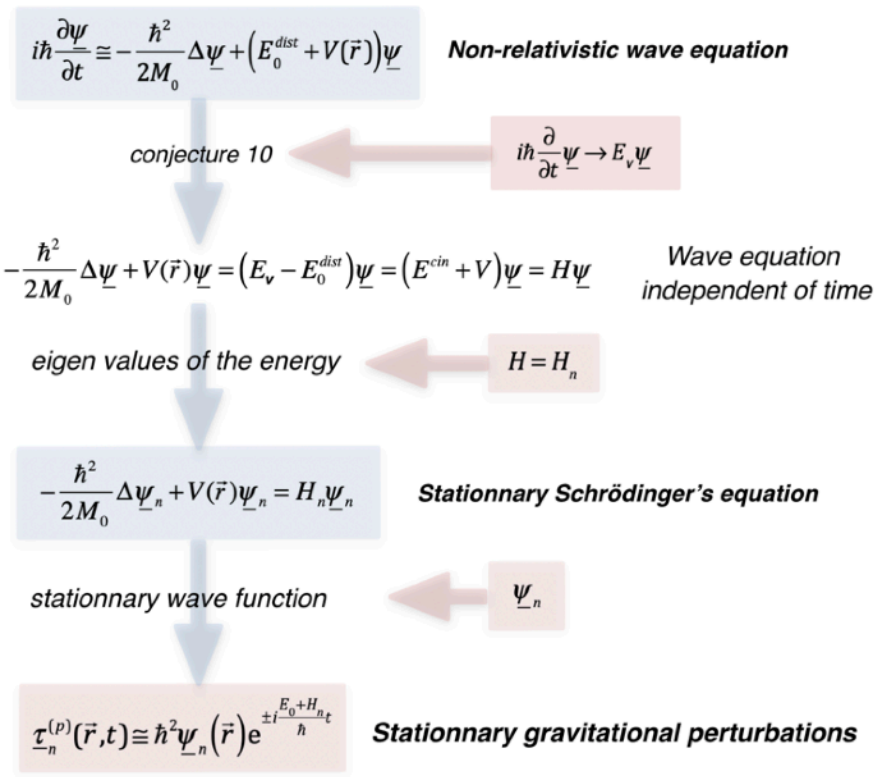

Figure 11.4 *- Schrödinger's standing wave equation with eigenvalues for a massive singularity placed in a static potential*

Here we find the expression of *the stationary Schrödinger's equation* of quantum physics, which is, as we know in quantum physics, a *problem with eigenvalues*, which means that the

Hamiltonian, that is say the sum of the potential energy and the kinetic energy associated with the movement of the singularity, given by $H = E^{cin} + V = E_{\nu} - E_0^{dist}$, is a constant which can take various eigenvalues H_n according to the potential $V(\vec{r})$, so that the wave function has eigenstates $\underline{\psi}_n$ satisfying the Schrödinger's equation with *eigenvalues* reported in figure 11.4.

On the basis of the solution $\underline{\psi}_n(\vec{r})$ of the wave equation, we deduce the true physical quantity, namely *the stationary expansion perturbations* associated with the singularity plunged in the stationary potential $V(\vec{r})$, given thanks to the relation of figure 11.3 by the expression $\underline{\tau}_n^{(p)}(\vec{r},t) \cong \hbar^2 \underline{\psi}_n(\vec{r}) \exp(\pm i(E_0 + H_n)t/\hbar)$, and it is the real part of $\underline{\tau}_n^{(p)}(\vec{r},t)$ which will represent the real stationary expansion perturbations. This solution for stationary gravitational perturbations is then presented in the form of oscillations of fixed pulsation $|\omega_f| = (E_0 + H_n)/\hbar$ dependent on the eigen energy H_n, oscillations which are modulated by a stationary envelope $\hbar^2 \underline{\psi}_n(\vec{r})$ dependent on the potential $V(\vec{r})$ via the Schrödinger's stationary wave equation.

About the interpretation of the wave function of gravitational fluctuations

The power of the dynamic Schrödinger equation (figure 11.3) and the stationary Schrödinger equation (figure 11.4) is well known in quantum physics. Many consequences linked to Schrödinger's equations are obviously directly applicable in the case of our approach.

For example, *the operator commutators* and *the Heisenberg uncertainty principles* are deduced directly using the conjecture 10. Thus, the quantum operator of momentum $\hat{p}_k = -i\hbar\partial/\partial\xi_k$ does not commute with the position operator $\hat{\xi}_k = \xi_k$ of a particle, because the commutator $\left[\hat{p}_k, \hat{\xi}_k\right] = \hat{p}_k \bullet \hat{\xi}_k - \hat{\xi}_k \bullet \hat{p}_k = -i\hbar$ is not zero. Likewise, the energy operator $\hat{E} = i\hbar\partial/\partial t$ does not commute with the time operator $\hat{t} = t$ since the commutator $\left[\hat{E}, \hat{t}\right] = \hat{E} \bullet \hat{t} - \hat{t} \bullet \hat{E} = i\hbar$ is not zero. These two commutating relations are expressions of *the Heisenberg uncertainty principle*, which says that the measurements of certain pairs of observables disturb each other, so that the measurement uncertainties $\Delta\xi_k$, Δp_k, Δt, ΔE, … are linked together by uncertainty relations.



Werner Heisenberg (1901-1976)

Another example is the calculation of *the stationary eigenstates of a particle* in different types of potentials (harmonic oscillator, anharmonic oscillator, particle in a box, rotation of two linked particles, particle in a central potential, etc.), or the calculation of *the density of states in phase space*, and many others that are beyond the scope of this book, but found in all books dealing with quantum physics.

From this perfect correspondence between our approach of gravitational perturbations associated with mobile topological singularities and the Schrödinger wave equation of quantum physics, experimentally very well verified, we deduce *à posteriori* that our conjecture 10 turns out to be absolutely founded. Therefore, our *"classical"* interpretation of quantum physics, namely that quantum physics follows from Newton's second partial equation of the cosmic lattice, is undoubtedly correct, so that Newton's equation of the cosmic lattice seems more and more play a crucial role *at the heart of all known theories of the Universe*.

Even though the complex wave functions $\underline{\psi}(\vec{r},t)$ do not give any indication concerning the position or the trajectory of the singularity, we can still find them a very interesting physical interpretation. As these wave functions correspond to *a complex representation of the amplitude and phase of the gravitational fluctuations of pulsation* $\underline{\omega}_f(\vec{r},t)$ *associated with the singularity*, it is entirely logical and probable that if there is locally no gravitational fluctuations, that is to say if the wave function is very small in certain places in space, there will be practically no chance of finding the topological singularity there, whereas if these fluctuations become maximum in some other places, there is a very good chance of finding the topological singularity there.

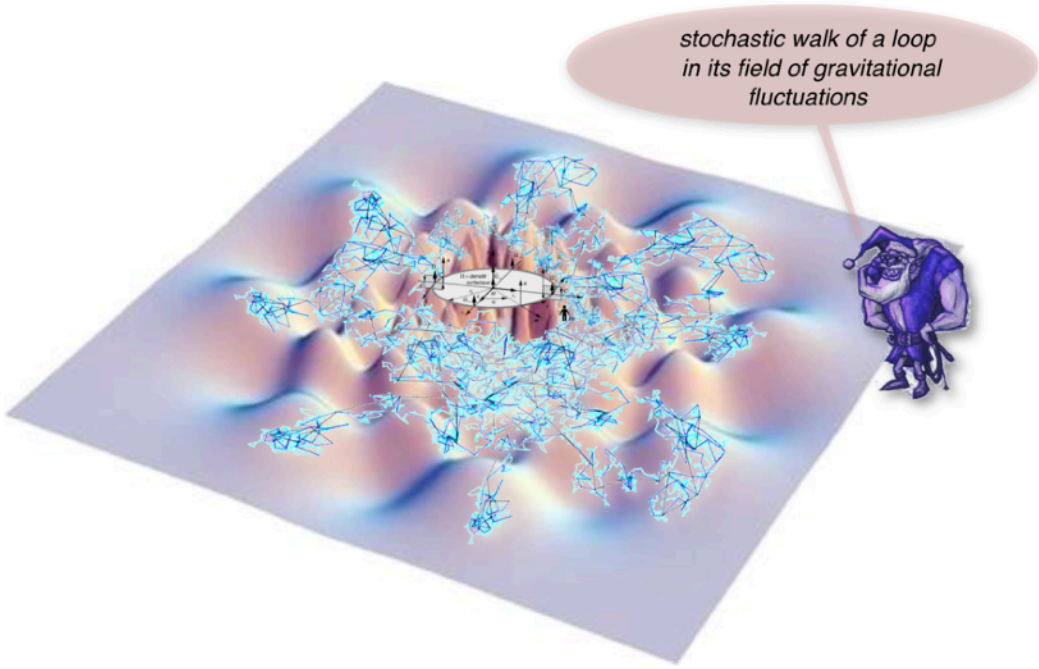

Figure 11.5 *- Illustration of the "stochastic march" of a singularity loop in its stationary field of gravitational perturbations*

We therefore come to a very interesting interpretation of this complex wave function $\underline{\psi}(\vec{r},t)$: it must certainly be associated with the probability of the presence of the topological singularity with which the gravitational fluctuations $e^{i\underline{\omega}_f t}$ are associated. The function $\underline{\psi}(\vec{r},t)$ is in fact a complex mathematical object representing the amplitude and the phase of the gravitational fluctuations $e^{i\underline{\omega}_f t}$, while a probability of presence is a positive scalar mathematical object whose sum over all the space must be equal to 1. Consequently, one possibility of extracting a quantitative value of probability of presence of the topological singularity from the function $\underline{\psi}(\vec{r},t)$ is to use the fact that the square of an oscillating function does have a positive scalar. In the case of a complex quantity like $\underline{\psi}(\vec{r},t)$, it is the product $\underline{\psi}(\vec{r},t) \cdot \underline{\psi}^*(\vec{r},t)$ of the complex function $\underline{\psi}(\vec{r},t)$ by its complex conjugate $\underline{\psi}^*(\vec{r},t)$ which represents the square of the amplitude of the function. It is therefore enough to normalize the product $\underline{\psi}(\vec{r},t) \cdot \underline{\psi}^*(\vec{r},t)$

taken over a portion ΔV of space by this product taken over all the space V likely to contain the singularity to obtain the probability P of finding the singularity in the portion ΔV of space, as illustrated in figure 11.5 . We therefore find the usual simple interpretation of the wave function in quantum physics, while giving it a conceptual explanation here.

The fact that the complex wave function $\underline{\psi}(\vec{r},t)$ makes it possible to deduce, not the position of the singularities at a given instant, but their probability of presence in a given place and at a given instant, also means that the wave equations, which allow one to compute $\underline{\psi}(\vec{r},t)$ or $\underline{\psi}_n(\vec{r})$ in the stationary case, and which are in fact nothing other than *emanations of the Newton's equation of the lattice* applied to the gravitational fluctuations, are at the same time *a new form of equations of the dynamics of microscopic topological singularities within the lattice*.

As the exact microscopic movements of topological singularities are not accessible and predictable via their complex wave function $\underline{\psi}(\vec{r},t)$, but only the probability of the presence of singularities subject to a potential $V(\vec{r},t)$ or $V(\vec{r})$ can be obtained, the real movements of microscopic singularities within the cosmological lattice must probably be stochastic and chaotic movements.

One can imagine for example that random gravitational fluctuations (see chapter 14), but of pulsations different from the pulsation $\underline{\omega}_f(\vec{r},t)$ associated with the topological singularity, and which would appear and disappear in the vicinity of the singularity, could come to shake it enough by providing it random pulses. These impulses would then contribute to stochastic movements of the singularity. But as the stochastic walk of the singularity must also be strongly coupled with its proper gravitational fluctuations of frequency $\underline{\omega}_f(\vec{r},t)$, this stochastic walk should present a statistical distribution of presence which would manifest itself via the probability of presence deduced from the wave function.

There are in fact two observable physical phenomena which present strong analogies with such a “stochastic march” of topological singularities:

- in solids, dislocations can exhibit a microscopic stochastic course under the effect of random thermal fluctuations (due to phonons) which can shake them up. There then appears a stochastic movement of dislocations, called *Brownian movement*, as described for example in the article «*overview on dislocation-point defect interaction: the brownian picture of dislocation motion*» [1].

- recent macroscopic experiments carried out in the laboratory with bouncing droplets on a vibrated liquid surface present fairly surprising results. The droplets move randomly on the liquid surface, which is why they have been called “walking drops” [2]. This “walk” is attributed to a resonant interaction of the gout with its own wave field [3]. The measurement of the probability of distribution of the drop on a limited liquid surface can then have a regularity presenting

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[1] *G. Gremaud, Materials Science and Engineering A 370 (2004) 191-198*

[2] *Stéphane Perrard: “Une mémoire ondulatoire: états prores, chaos et probabilités”, thèse de doctorat, 2014, Université Paris Diderot (https://tel.archives-ouvertes.fr/tel-01158368)*

[3] *D. M. Harris, J. Moukhtar, E. Fort, Y. Couder, J. W. M. Bush: “Wavelike statistics from pilot-wave dynamics in a circular corral” Physical Review E, 88, 011001(R), 2013*

astonishing similarities with the probability of the presence of quantum microscopic particles confined in a well of potential [4].

In fact, the interpretation of quantum physics that emerges from our approach *comes close enough to the interpretation of Bohm, in its stochastic version, introduced in 1954* as a development of *the pilot wave theory of Louis de Broglie of 1927*. Indeed, in our approach, the wave function $\underline{\psi}(\vec{r},t)$ is a real and objective field, corresponding to the envelope of the gravitational fluctuations of the volume expansion of the cosmological lattice associated with topological singularities, while topological singularities are also, as for them, real particles (dislocation or disclination loops) which at all times have real coordinates in space and a given momentum. These are indeed the main ideas of Bohm's interpretation. And the image of the world conveyed by our approach is clearly indeterministic as is the Bohmian image of quantum physics, in the sense that we do not have direct access to the positions of the singularities, but only to the notions of probabilities that we deduce from the wave function, which are only a reflection of our ignorance of the underlying history which determines the course of events for microscopic topological singularities.


Louis de Broglie
(1892-1987)


David Bohm
(1917-1992)

Superposition of topological singularities, bosons, fermions and exclusion principle

One can legitimately wonder what becomes of the field of gravitational fluctuations when two topological singularities are close to each other. Imagine that we consider two singularities *(a)* and *(b)* which evolve in the same space, and therefore in the same potential $V(\vec{r})$. Let's look for the superimposed standing wave function, that is, how to write the volume expansion perturbations due to the two singularities at the same time. Assuming that in the state of stationary superposition, the stationary Schrödinger equations remain valid for the two singularities (figure 11.6), let us try to combine these two relations, by multiplying the first by $\underline{\psi}_m(\vec{r}_b)$ and the second by $\underline{\psi}_n(\vec{r}_a)$, and by summing the whole. We get a new equation which is nothing other than the Schrödinger equation for the superposition wave function $\underline{\psi}_n(\vec{r}_a)\underline{\psi}_m(\vec{r}_b)$. We therefore deduce that the oscillatory perturbations of the volume expansion due to the superposition of the two singularities are written as a product of the two wave functions multiplied by the oscillations $\exp\left(\pm i(E_0+H_n)t/\hbar\right)$ and $\exp\left(\pm i(E_0+H_m)t/\hbar\right)$. We note that there are then two types of possible superpositions, according to the signs of exponential exponents, which have very different global oscillation frequencies, related to the sum and the difference of the energies of the singularities, namely $\exp\left(\pm i(2E_0+H_n+H_m)t/\hbar\right)$

[4] *R. Brady, R. Anderson: "Why bouncing droplets are a pretty good model of quantume mechanics", University of Cambridge Computer Laboratory, 2014 (arXiv:1401.4356v1)*

and $\exp\left(\pm i(H_n - H_m)t/\hbar\right)$.

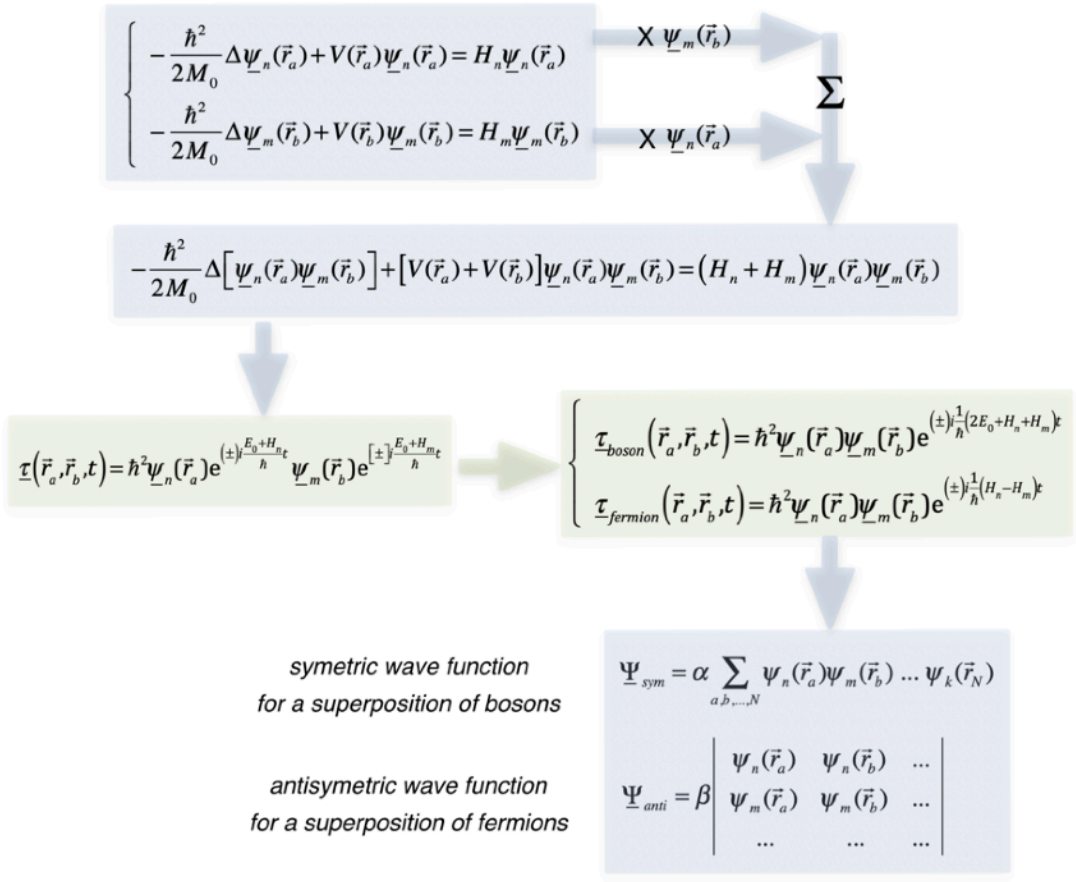

Figure 11.6 *- Stationary equation for the superposition of topological singularities, gravitational fluctuations of bosons and fermions and symmetrical and anti-symmetrical wave functions*

By analogy with quantum physics, we will call *bosons* the singularities corresponding to the first superposition solution, whose pulsation is $\left(2E_0 + H_n + H_m\right)/\hbar$, because the two singularities can occupy the same energy level without disappearance of the oscillatory perturbations of the expansion. As for the singularities which correspond to the second pulsation solution $\left|H_n - H_m\right|/\hbar$, they will be called *fermions* because they cannot be superimposed in the same energy level since in this case the oscillatory gravitational perturbations of the expansion disappear.


Wolfgang Pauli (1900-1958)

This observation on the way that the singularities are superimposed reveals therefore directly the famous *principle of exclusion of Pauli*: the singularities which combine according to

the second possibility, namely the *fermions*, cannot be found in the same state of energy.

In usual quantum physics, where we only describe the wave functions, and where we ignore the real physical meaning of these wave functions in terms of amplitude and phase of oscillatory gravitational perturbations, we can show this difference between bosons and fermions directly in the superposition wave function $\underline{\psi}_n(\vec{r}_a)\underline{\psi}_m(\vec{r}_b)$. For this, we note that for the same value of the energy E of the system, there are two possible solutions of the equation for the superposition wave function $\underline{\Psi}$, which simply correspond to exchanging the two identical singularities, let $\underline{\Psi} = \underline{\psi}_n(\vec{r}_a)\underline{\psi}_m(\vec{r}_b)$ and $\underline{\Psi} = \underline{\psi}_n(\vec{r}_b)\underline{\psi}_m(\vec{r}_a)$. Now, one of the fundamental properties of linear and homogeneous differential equations is that any linear combination of particular solutions of this type of equation is also a solution, so that the most general solution of the Schrödinger equation can be written as a a superposition $\underline{\Psi} = \alpha\underline{\psi}_n(\vec{r}_a)\underline{\psi}_m(\vec{r}_b) + \beta\underline{\psi}_n(\vec{r}_b)\underline{\psi}_m(\vec{r}_a)$. This expression would seem to indicate that there exists a large number of stationary states for a system of two singularities. However, it must now be taken into account that, because of the principle of uncertainty linked to the operator commutators, identical singularities lose their individuality. We say that identical singularities are indistinguishable, which quite simply means that it is not possible to follow the trajectory of a given singularity over time. If we consider the wave function $\underline{\Psi}$ of the system, we know that $\underline{\Psi}^2$ determines the probability of finding the two singularities in a certain portion of space. If we exchange the two singularities, it is clear that $\underline{\Psi}^2$ must remain unchanged. On the other hand, the phase of $\underline{\Psi}$ can be modified by this exchange, so that $\underline{\Psi} \to \underline{\Psi}e^{i\eta}$. If we proceed to a second exchange of singularities, we obviously have $\underline{\Psi} \to \underline{\Psi}e^{2i\eta}$ and we find ourselves in the initial state $\underline{\Psi}$, so that $e^{2i\eta} = 1$. To satisfy this last condition, it suffices that $e^{2i\eta} = +1$ or $e^{2i\eta} = -1$.

In the case where the wave function $\underline{\Psi}$ is transformed into $\underline{\Psi} \to +\underline{\Psi}$ during the exchange of the two singularities, the wave function is said to be *symmetrical*, and the singularities are called *bosons*. The wave function is written $\underline{\Psi} = \alpha\left(\underline{\psi}_n(\vec{r}_a)\underline{\psi}_m(\vec{r}_b) + \underline{\psi}_n(\vec{r}_b)\underline{\psi}_m(\vec{r}_a)\right)$, where α is a normalization factor.

If the wave function $\underline{\Psi}$ changes to $\underline{\Psi} \to -\underline{\Psi}$, the wave function is said to be *antisymmetric*, and the singularities are called *fermions*. The wave function is written with a normalization factor β like $\underline{\Psi} = \beta\left(\underline{\psi}_n(\vec{r}_a)\underline{\psi}_m(\vec{r}_b) - \underline{\psi}_n(\vec{r}_b)\underline{\psi}_m(\vec{r}_a)\right)$.

The indistinguishability of the two singularities then emerges clearly from the two previous expressions of the wave function $\underline{\Psi}$. We also note that, for the antisymmetric wave function, it is not possible that the two singularities are in the same state since $\underline{\Psi}$ would then be zero: this is indeed the mathematical expression at the level of the wave function itself of *the principle of exclusion of Pauli,* who says that two fermions cannot occupy the same state simultaneously because the gravitational perturbations disappear in this case.

In the case of a system of N identical singularities, the preceding concepts are easily generalized. In the case of bosons, the symmetric wave function $\underline{\Psi}_{sym}$ of the system is written in the form $\underline{\Psi}_{sym} = \alpha\sum\psi_n(\vec{r}_a)\psi_m(\vec{r}_b)\ldots\psi_k(\vec{r}_N)$, where the sum relates to the possible permutations of all the different states of the system. If the system has n_1 singularities in the energy state n, n_2 singularities in the energy state m, n_3 singularities in the energy state k, etc., the number of terms making up the wave function $\underline{\Psi}_{sym}$ is given by all the possible permutations, namely $P = N!/\left(n_1!n_2!n_3!\ldots\right)$.

In the case of fermions, the antisymmetric wave function $\underline{\Psi}_{anti}$ of the system can be written

in the form of a determinant, as shown in figure 11.6. Indeed, the permutation of two columns of a determinant changes the sign of the determinant, which ensures the anti-symmetry of the wave functions under the exchange of two of the singularities. On the other hand, we also know that a determinant is zero if two lines are identical, which here corresponds to the expression of *the Pauli exclusion principle*, namely that a given state cannot be occupied by more than one fermion.

Demystifying quantum physics

It is quite remarkable that the wave function associated with the gravitational perturbations of the volume expansion is perfectly similar to the quantum wave function of a particle, and that it satisfies a wave equation identical to Schrödinger's equation. This obviously deserves further discussion.

We have shown in this chapter that *beyond the quantum decoherence limit*, i.e. for singularities whose mass density is above a certain critical value (figure 7.0), it is possible to link to this singularity a relativistic wave equation (figure 11.1) deduced by a "subterfuge" from Newton's second partial equation (figure 5.1). This equation made it possible to describe *the dynamic "gravitational" fluctuations of expansion* associated with a massive singularity moving at relativistic velocities within the network (figure 11.2). As for the non-relativistic wave equation of a singularity bound by a potential, it is absolutely identical to *the Schrödinger equation of quantum physics* (figure 11.3) since their respective interpretations in terms of the probability of the presence of a particle are *identical*. The key passages used to arrive at the Schrödinger equation of a singularity from Newton's second partial equation for gravitational expansion perturbations are, firstly, *the conjecture 10* postulating *the physical meaning of time and space operators applied to the wave function*, and secondly *the "reduction" of the wave equation* from the second-degree wave equation into spatial derivatives to the first-degree wave equation into spatial derivatives, again using *conjecture 10*. It is these two key passages that have made it possible to establish a physical theory quite similar to quantum physics to describe the microscopic behaviour of topological singularities within a cosmological network that does not exhibit longitudinal waves.

But our approach still lacks a deep physical explanation of these two key passages and their reason for being. In particular, we can legitimately ask the question of why the Planck constant exists, where its value comes from, and whether it is really a universal constant or if it is deductible from the other constants appearing in our approach. An answer to these questions would allow an even more in-depth and definitive understanding of quantum physics.

But in our approach, the complex wave function $\underline{\psi}\left(\vec{r},t\right)$ and the Schrödinger wave equation are still physically demystified, since they become there the mathematical expressions of the envelope and of the phase of the vibratory fluctuations of the lattice expansion, therefore *gravitational fluctuations* correlated with topological singularities.

From this completely innovative interpretation of quantum physics, the possibility of having *“bosons”* and *“fermions”* type singularities, the fact that there is *indistinguishability* between topological singularities when they contribute to the same field of correlated gravitational fluctuations, and the fact that singularities of the "fermion" type must satisfy an exclusion principle similar to *the Pauli exclusion principle* is probably the most remarkable and astonishing

point of our calculations, because it demystifies a side that has always been most obscure in quantum physics.

Finally, it is equally remarkable to note that all these properties, such as for example *the properties of superposition* (the symmetry of the wave function $\underline{+}\Psi_{sym}$ of singularities of bosons type and the anti-symmetry of the wave function $\underline{+}\Psi_{antisym}$ of singularities of fermions type, the *indistinguishability* of topological singularities and the principle of exclusion) are direct consequences of the fact that the gravitational fluctuations associated with one or more singularities must satisfy Newton's second partial equation of the cosmological network.

In fact, the image of a field of vibrational gravitational fluctuations correlated with a topological singularity has enormous potential to explain simply observed and / or calculated quantum phenomena, but still remains very mysterious in the framework of usual quantum physics. Just think of the following few examples:

- the concept of wave-particle duality of quantum physics, which finds an immediate and simple explanation here since the particle is the topological singularity, and that the wave is the field of the gravitational fluctuations which is correlated to it,
- the experiments of quantum interference obtained by the passage of particles through two slits, but by letting pass only one particle at a time, which can very well be explained now by the fact that each topological singularity has actually to go through only one of the slits. But the field of gravitational fluctuations associated with it crosses the two slits, hence the possibility of interference of these fluctuations at the exit of the two slits, resulting in their coupling with the singularity a modification of its trajectory, and finally a statistical distribution of the successive impact points of the particles on the screen placed after the two slits,
- the Heisenberg's uncertainty principle, which is obviously satisfied in our approach since it admits a fortiori the same interpretation of the operators acting on the wave function as quantum physics, and therefore satisfies all the relations of classical quantum physics . The uncertainty relationships are then directly linked to the existence of gravitational perturbations correlated to the singularity,
- the very mysterious experiments of quantum entanglement and quantum decoherence, since one can very well imagine that the entanglement is the fact for two or more topological singularities to have in common a single field of gravitational fluctuations, in which case the fact of acting on only one of the singularities will modify this common field of gravitational fluctuations, which will necessarily act on the other singularities which participate in the entanglement, and which can also cause quantum decoherence, in other words the decoupling of the topological singularities involved in the common field of gravitational fluctuations.

Einstein said *"God does not play dice"* when talking about quantum physics, meaning that the quantum physics of his time was not a complete theory, and that there had to be a rational and pragmatic explanation for the probabilistic aspect of quantum theory. This opinion of Einstein has been strongly contested, not to say highly criticized. It has been demonstrated that there could not be local hidden variables to explain quantum physics, but on the other hand nothing prevents there from being non-local hidden variables, and this is precisely the case of gravitational fluctuations correlated to topological singularities. It is therefore clear here that Einstein was indeed right, and that there is indeed a completely rational explanation of quantum physics.

There is, moreover, a highly ironic note in the famous Einstein sentence, since quantum physics would be explained by gravitational fluctuations of the volume expansion field and by a stochastic movement of topological singularities interacting with these gravitational fluctuations. And these are precisely the ingredients with which God, if he existed, could play dice, and that, oh ironic irony, it is precisely Einstein himself who is the inventor of the explanations of the General Gravitation and the Brownian Motion, which earned him the Nobel Prize in physics.

Thus, our explanation of quantum physics proves Einstein right, by showing that quantum physics is the expression of gravitational fluctuations at very small scales in a cosmological lattice without propagation of longitudinal waves. Consequently, all modern attempts to quantify gravitation are bound to fail since quantum physics is precisely the expression of dynamic gravitational fluctuations on a microscopic scale.